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Claus Müller

Foundations of the Mathematical Theory of Electromagnetic Waves

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Prof. Dr. Claus Müller
Technische Hochschule Aachen

Revised and enlarged translation of
Grundprobleme der mathematischen Theorie
elektromagnetischer Schwingungen, 1957
(Grundlehren der mathematischen Wissenschaften, Vol. 88)

In cooperation with Dr. T. P. Higgins

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Preface

The technical applications of the electromagnetic waves created a field of research similar to the classical theory of the Newtonian potential which aims at a mathematical theory of the electromagnetic waves.

This trend was initiated by the strongly mathematical character of the fundamental papers published by *H. Hertz* and *G. Heaviside* between 1880 and 1890. Their presentation of *Maxwell's* theory formulated many mathematical problems of great generality and reduced the theoretical description of the electromagnetic phenomena to the solution of well defined mathematical problems.

The rapid technical development of the electromagnetic waves began at the time when the Dirichlet and Neumann problems of potential theory were first solved. Following *Fredholm's* paper of 1904 on linear integral equations many of the open questions of mathematical physics were settled in quick succession by *D. Hilbert* and *H. Poincaré*.

It seems natural that these results among which the boundary value and the eigenvalue problems are best known influenced the theory of electromagnetic waves. The first mathematical investigations are therefore closely related to the classical potential theory. The most interesting results of this time are the formulations of the Lorentz postulate regarding the asymptotic behavior of the eigenfrequences of cavities which *H. Weyl* gave between 1910 and 1915. Here it became obvious that the problems of the theory of electromagnetic waves can not be understood as simple extensions of the problems of potential theory, but that they possess typical difficulties which result from the special form of *Maxwell's* equations.

In analogy to the techniques of potential theory, methods were developed which, following the idea of the separation of variables, discovered special solutions of *Maxwell's* equations. Thus *G. Mie* solved the problem of the diffraction by a sphere in 1908. The diffraction by the wedge and half-plane which *A. Sommerfeld* found at the turn of the century uses related structures.

It was in *Sommerfeld's* papers that the essential difference from potential theory was first seen. Then it was noted that the electromagnetic waves have a peculiar behavior at infinity which could not be expected from the results of potential theory. For all problems related to the propagation of electromagnetic waves in an infinite medium, difficulties occur which can not be treated in analogy to potential theory.

During the last decades it became clear that the asymptotic behavior at infinity is a structure of decisive importance for the theory of electromagnetic waves. *A. Sommerfeld* discovered in 1898 that the uniqueness of the scalar diffraction problem for acoustical waves is guaranteed only if radiation conditions are imposed. These conditions may be considered as the mathematical formulation of the fact that the transport of energy is directed towards infinity. The radiation condition is thus a sort of boundary condition at infinity. It is only with this radiation condition that a consistent mathematical formulation of the scalar diffraction problems is possible.

A rigorous mathematical treatment of these problems started as late as 1943 when *F. Rellich* proved the uniqueness of the exterior boundary value problems for the reduced wave equation. The existence of solutions was proved in 1953 by *H. Weyl* and the author. In the Soviet Union similar results were obtained by *W. D. Kupradse.*

It is now possible to treat the fundamental problems of the theory of electromagnetic waves with the rigor and generality which is required of a mathematical theory. I intended to develop a mathematical theory which serves as a critical dialogue between physical intuition and mathematical formulation in the same sense as does the theory of the Newtonian potential.

The first German edition of 1957 gave a self consistent presentation of my contributions to the theory of electromagnetic waves since 1945. Some results, in particular the studies on radiation patterns, appeared there for the first time.

This second edition is a revised translation of the first edition with minor modifications and corrections. The theories of spherical harmonics and Bessel functions are indispensable tools; both subjects are presented to the extent which is needed for the theory of electromagnetic waves. Another topic which is often needed is the theory of vector fields on closed surfaces with its implications of differential geometry and topology. The theory of linear operators in the sense of functional analysis yields the existence proofs for the boundary value and diffraction problems. It is therefore a very important part of a mathematical theory. These subjects are presented here in a self-consistent way.

Due to the complexity of the structure and the many different concepts and results, a homogeneous theory is difficult to achieve. I have tried a presentation which shows the theory of electromagnetic waves as a leading theme of mathematical theories.

In an introductory chapter the Maxwell Hertz theory of electromagnetic waves is briefly exposed. The integral form of the equations is taken as the basis of an extended interpretation of these laws by means of new formulations of the basic operations of vector analysis.

The first chapter gives a theory of vector analysis with proofs of well-known identities under conditions which are adapted to the requirements of the theories to be expounded.
known identities under conditions which are adapted to the requirements of the theories to be expounded.

The following chapters contain a theory of the reduced wave equation with emphasis on the asymptotic laws at infinity.

After these preparations the basic problems of the mathematical theory of electromagnetic waves can be studied. The radiation condition for electromagnetic waves plays an important part in these results.

Chapter VI gives the theory of diffraction. This problem is treated for the case of a diffracting object with continuously varying material properties and then for the case of diffraction by a homogeneous object. A discussion of the perfect reflection, which was not contained in the first edition, is added. This leads to the theory of boundary value problems.

In a closing chapter the topological properties of radiation patterns are investigated. Here information is gained on the asymptotic polarization of electromagnetic waves at infinity.

I did not intend to give a complete and exhaustive treatment of the many problems and methods of the mathematical theory of electromagnetic waves, but I have tried to show the relevance and harmony of the basic ideas as given by *Maxwell's* equations, *Huygens's* principle and the radiation conditions. They form the basis of a theory which is as consistent as the classical potential theory and which offers a greater variety of problems and structures.

I am greatly indebted to Dr. T. P. Higgins for much help with the preparation of the English edition and for valuable comment. For substantial help with the proof I wish to thank Dr. C. Engeln. It is also a pleasure to acknowledge the ready cooperation of the publisher in solving the technical problems of printing this book.

Brissago, August 1969 CLAUS MÜLLER

Contents