Springer Series in

SOLID-STATE SCIENCES 140

Springer-Verlag Berlin Heidelberg GmbH

Physics and Astronomy ONLINE LIBRARY

http://www.springer.de/phys/

Springer Series in
SOLID-STATE SCIENCES

The Springer Series in Solid-State Sciences consists of fundamental scientific books prepared by leading researchers in the field. They strive to communicate, in a systematic and comprehensive way, the basic principles as well as new developments in theoretical and experimental solid-state physics.

126 **Physical Properties of Quasicrystals**
Editor: Z.M. Stadnik

127 **Positron Annihilation in Semiconductors**
Defect Studies. By R. Krause-Rehberg and H.S. Leipner

128 **Magneto-Optics**
Editors: S. Sugano and N. Kojima

129 **Computational Materials Science**
From Ab Initio to Monte Carlo Methods
By K. Ohno, K. Esfarjani, and Y. Kawazoe

130 **Contact, Adhesion and Rupture of Elastic Solids**
By D. Maugis

131 **Field Theories for Low-Dimensional Condensed Matter Systems**
Spin Systems and Strongly Correlated Electrons
By G. Morandi, P. Sodano, A. Tagliacozzo, and V. Tognetti

132 **Vortices in Unconventional Superconductors and Superfluids**
Editors: R.P. Huebener, N. Schopohl, and G.E. Volovik

133 **The Quantum Hall Effect**
By D. Yoshioka

134 **Magnetism in the Solid State**
By P. Mohn

135 **Electrodynamics of Magnetoactive Media**
By I. Vagner, B.I. Lembrikov, and P. Wyder

136 **Nanoscale Phase Separation and Colossal Magnetoresistance**
The Physics of Manganites and Related Compounds. By E. Dagotto

137 **Quantum Transport in Submicron Devices**
A Theoretical Introduction. By W. Magnus and W. Schoenmaker

138 **Phase Separation in Soft Matter Physics**
Micellar Solutions, Microemulsions, Critical Phenomena
By P.K. Khabibullaev and A.A. Saidov

139 **Optical Response of Nanostructures**
Microscopic Nonlocal Theory By K. Cho

140 **Fractal Concepts in Condensed Matter Physics**
By T. Nakayama and K. Yakubo

Series homepage – http://www.springer.de/phys/books/sss/

Volumes 1–125 are listed at the end of the book.

T. Nakayama K. Yakubo

Fractal Concepts in Condensed Matter Physics

With 72 Figures

Springer

Professor Tsuneyoshi Nakayama
Professor Kousuke Yakubo
Department of Applied Physics
Graduate School of Engineering
Hokkaido University
Sapporo 060-8628
Japan

Series Editors:

Professor Dr., Dres. h. c. Manuel Cardona
Professor Dr., Dres. h. c. Peter Fulde*
Professor Dr., Dres. h. c. Klaus von Klitzing
Professor Dr., Dres. h. c. Hans-Joachim Queisser
Max-Planck-Institut für Festkörperforschung, Heisenbergstrasse 1, D-70569 Stuttgart, Germany
* Max-Planck-Institut für Physik komplexer Systeme, Nöthnitzer Strasse 38
D-01187 Dresden, Germany

Professor Dr. Roberto Merlin
Department of Physics, 5000 East University, University of Michigan
Ann Arbor, MI 48109-1120, USA

Professor Dr. Horst Störmer
Dept. Phys. and Dept. Appl. Physics, Columbia University, New York, NY 10027 and
Bell Labs., Lucent Technologies, Murray Hill, NJ 07974, USA

ISSN 0171-1873
ISBN 978-3-642-05711-3 ISBN 978-3-662-05193-1 (eBook)
DOI 10.1007/978-3-662-05193-1

Library of Congress Cataloging-in-Publication Data: Nakayama, T. (Tsuneyoshi), 1945–. Fractal concepts in condensed matter physics/ T. Nakayama, K. Yakubo. p. cm. – (Springer series in solid-state sciences, ISSN 0171-1873 ; 140). Includes bibliographical references and index. (alk.paper) 1. Condensed matter–Mathematics. 2.Fractals. I. Yakubo, K. (Kousuke) II. Title. III. Series. QC173.458.F72N35 2003 530.4'1'01514742–dc21 2003050426

http://www.springer.de

Originally published by Springer-Verlag Berlin Heidelberg New York in 2003.
Softcover reprint of the hardcover 1st edition 2003

Data conversion: Le-TeX, Leipzig
Cover concept: eStudio Calamar Steinen
Cover production: *design & production* GmbH, Heidelberg

Printed on acid-free paper SPIN: 10782866 57/3141/ba - 5 4 3 2 1 0

Preface

This book is written with the intention of presenting a systematic description of the underlying concept of fractals in a range of topics in condensed matter physics. The idea of fractals is based on self-similarity, which is a symmetry property of a system characterized by invariance under an isotropic scale transformation. This concept can be used to build simple pictures of the realm of nature. In Chaps. 1 and 2, we have given a brief survey of typical examples of fractal structures. We have also included a concise account of methods for calculating fractal dimensions characterizing their fractalities.

The dynamical properties of fractal structures constitutes the first major part of this book, including spectral densities of states, transport and localization/delocalization of waves. Chapter 3 contains basic results on percolation theory, including the introduction of various exponents characterizing percolating networks. The notion of percolation satisfactorily describes a large number of dynamic phenomena observed in fractal structures, such as gelation processes, transport in amorphous materials, hopping conduction in doped semiconductors, and many other applications. In addition, it forms the basis for studies of the flow of liquids or gases through porous media. For the analysis of these dynamic properties, the problem of diffusion on fractal structures plays a key role. In Chap. 5, we have tried to give a complete description of all standard results on anomalous diffusion, including its relevance to the dynamics of fractal networks. The results in this chapter are applied to the dynamic problems of fractal networks in Chap. 6. We include here many basic results on the dynamics obtained via large-scale numerical simulations. Some classes of diluted Heisenberg magnets take the geometrical structures of percolating networks. Spin waves in diluted Heisenberg magnets should reflect their fractalities. This subject is relatively new. Chapters 7 and 8 contain the dynamic scaling arguments on this subject as well as the important results obtained by large-scale numerical simulations.

Another central issue in this book is the concept of multifractals. Physical quantities often distribute in a complex manner on fractal or non-fractal (Euclidean) supports. Multifractals are currently used to describe such complex distributions. The idea of multifractals, first introduced to analyze energy dissipation in turbulent flow, has widened our view of intricate distributions observed in various fields of science. In particular, multifractal analysis provides a number of significant insights into condensed matter physics, such as current distributions in fractal networks, the growth dynamics of diffusion-limited aggregations and viscous fingerings, crystallization on bilayer films, and energy spectra of quasicrystals. Among these, we have paid special attention to the relevance of multifractals in quantum critical phenomena, e.g., the multifractal property of electron wavefunctions at the metal–insulator Anderson transition. Multifractal analysis is a standard method for studying quantum critical properties of the Anderson transition. In Chap. 4, we have tried to familiarize the reader with various computational techniques using multifractal exponents, which are currently used in multifractal analysis. An entire spectrum of exponents characterizing multifractality at the transition point can be used to identify the universality class of the system. Chapters 9 and 10 describe the Anderson transition from the multifractal standpoint.

Our primary concern was to make this book as self-contained as possible and we hope that this purpose has been achieved by the above arrangements.

Sapporo
January 2003

Tsuneyoshi Nakayama
Kousuke Yakubo

Acknowledgements

We are particularly grateful to Professor Raymond Orbach, who has investigated together on several subjects described in this book. We are also indebted to stimulating discussions with the late Professor Shlomo Alexander. Various parts of the manuscript are benefitted from conversations with Professors Hideo Aoki, Yshai Avishai, Eric Courtens, the late Hironubo Ikeda, Mitsugu Matsushita, Tomi Ohtsuki, Keith Slevin, and Takamichi Terao, and we would like to thank them for their contributions. We wish to appreciate the patient and kind encouragement of Dr. C. E. Ascheron of Springer-Verlag for publication. Dr. T. Haard kindly supplied the computer simulated figure of aerogel. Finally, one of the Autors (T.N.) would like to thank Professor Peter Fulde for his hospitality during the stay at Max-Planck-Institute for the Physics of Complex Systems, where a part of this book was written in the summer of 2002.

The true end is not in the reaching of a limit, but in a completion which is limitless.

R. Tagore (1928)

Notation and Abbreviations

α	Lipschitz–Hölder exponent
μ_i	probability measure at the i th site
$\mu_{b(l)}$	box measure of a box b of size l
ν	correlation (or localization) length exponent
$\tau(q)$	mass exponent
ξ	correlation (or localization) length
D_f	fractal dimension
D_q	generalized dimension
$D(\omega)$	density of states
$\tilde{d}_{AF}$	spectral dimension of antiferromagnetic spin-wave fractons
$\tilde{d}_b$	spectral dimension for bending fractons
$\tilde{d}_s$	spectral dimension
$\tilde{d}_{st}$	spectral dimension for stretching fractons
$\tilde{d}_w$	diffusion exponent
$\overline{e_\lambda}$	average strain tensor
p_c	percolation threshold
z	dynamic exponent for vibrational fractons
z_{AF}	dynamic exponent for antiferromagnetic spin-wave fractons
$z(q)$	correlation exponent
1D	one dimension (one-dimensional)
2D	two dimensions (two-dimensional)
3D	three dimensions (three-dimensional)
BP	bond percolation
DID	dipole-induced dipole
DLA	diffusion-limited aggregation
DOS	density of states
GOE	Gaussian orthogonal ensemble
GSE	Gaussian symplectic ensemble
GUE	Gaussian unitary ensemble
HRN	hierarchical resistor networks
INS	inelastic neutron scattering
QHS	quantum Hall system
SLSP	single-length scaling postulate
SP	site percolation

Contents