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Philippe A. Martin    François Rothen

# Many-Body Problems and Quantum Field Theory

## An Introduction

Translated by Steven Goldfarb
With 102 Figures, 7 Tables and 23 Exercises

Springer

Professor Philippe A. Martin
Swiss Federal Institute for Technology
Institute for Theoretical Physics
1015 Lausanne
Switzerland

Professor François Rothen
University of Lausanne
Institute of Condensed Matter Physics
1015 Lausanne
Switzerland

*Translator*:
Dr. Steven Goldfarb
University of Michigan
CERN - EP
1211 Geneva 23
Switzerland

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# Foreword

This text is a revised and augmented version of a course given to graduate and Ph.D. students in the context of the doctoral school for physics in the French-speaking part of Switzerland. This doctoral school provides a common teaching program for the universities of Bern, Fribourg, Geneva, Neuchâtel and Lausanne, as well as for the Swiss Federal Institute of Technology in Lausanne. The scope of the course should be sufficiently general to interest both experimentalists and theoreticians wishing to engage in research in condensed matter or nuclear and particle physics. The prerequisites are an introductory course to quantum mechanics and elements of classical electromagnetism and statistical mechanics.

Our main concern was how to maintain a reasonably broad level of knowledge for students with different orientations, in a world of research where the price of survival is extreme specialization and competitiveness. Is it still possible in the available time to provide a cultural education in physics by relatively elementary means and in an optimized form? We believe that this is an essential pedagogical duty. Attempting to meet this challenge has determined the conception of this book: each individual part of it is standard and without novelty but should belong, in our opinion, to the basic culture of every physicist; only their common organization in a single house of decent size might possibly be put to our credit.

We have tried to keep a balance between formal developments and the physical applications: in fact they cannot be separated insofar as mathematical methods develop naturally under the necessity of resolving physical questions. Concerning the applications, we have always given a short description of the phenomenological context so that the main information about physical facts is available from the start without recourse to other sources. In the formal developments, we adopt the usual notation of physicists, while aiming at mathematical precision. The reader is warmly encouraged to improve his practice of the formalism by checking and reproducing for himself the algebra given in the text. Some more extended exercises are proposed at the end of each chapter in order to illustrate additional aspects not introduced in the main text.

For each of the systems discussed in this book, we have tried to exhibit how the main physical ideas can be captured in a formalized description by the appropriate tools. In this spirit several important branches of physics are represented: solid state physics (cohesion and dielectric properties of the electron gas, phonons and electron–phonon interactions), low temperature physics (superconductivity and superfluidity), nuclear physics (pairing of nucleons), matter and radiation (interaction of atoms with the quantum-electromagnetic field), particle physics (interaction by exchanged intermediate particles, mass generation by the Higgs mechanism).

These choices could be considered rather conservative, compared to topical new developing areas. However we think that they still serve as indispensable paradigms for the understanding of any more advanced subject. Also, in keeping with our aim of offering a broad formative view to our readers, they enable us to illustrate similarities and differences between concepts stemming from various domains in physics. In this respect, the first chapter presents a parallel exposition of classical electromagnetism and classical elasticity, with the purpose of introducing and comparing the notions of photon and phonon. Moreover, quantum fields (Chap. 8) cannot be understood without a good knowledge of their classical analogues. Chapter 2 is devoted to a simple description of collective effects due to Bose and Fermi statistics. Bose condensation is described and the role of Fermi statistics for the stability of matter and in astrophysical objects is discussed. In the third chapter we develop the so-called second quantized formalism in full generality without reference to any particular system, so that it will be available in any situation where the number of particles varies. Chapters 4, 5 and 6 are devoted to the use of the variational method. It is hoped that the reader will appreciate the wide range of applications of the idea of fermionic pairing formulated in the BCS theory for superconductivity (Chap. 5) as well as for nuclear matter (Chap. 6). The relationship between superfluidity (Chap. 7) and superconductivity on the one hand, and collective excitations of the nuclei on the other, is put into perspective. The quantum-electromagnetic field serves as a model for other quantized matter fields in Chap. 8. The concept of gauge theory is introduced and the close analogy between the Higgs mechanism and the Meissner effect displayed. The method of Feynman graphs is explained in Chaps. 9 and 10, stressing again the existence of a common language for condensed matter and field theory. We essentially give the physical interpretation of diagrams without performing the corresponding more technical quantitative calculations. The analysis is restricted to ground-state properties: non-zero temperature Green functions and the alternative functional integration viewpoint are not considered.

The book is not aimed at the specialist in any of the addressed topics. In fact, no chapter is intended to provide the up-to-date knowledge necessary for an immediate fight on the battlefront of research. We refer in particular to the present state of relativistic quantum field theory since, without mentioning electroweak theory and chromodynamics, no presentation of the Lorentz group or of the Dirac equation can be found here. From the viewpoint of condensed matter, high-$T_c$ is only briefly touched, mesoscopic physics and highly correlated fermions are not discussed. To the prospective particle physicist, the book can merely give a complementary education on the use of similar techniques in condensed-matter physics. Conversely, physicists belonging to the latter discipline, although they may be aware of the importance of field theory for particle physics, should learn about fundamental ideas underlying both domains. We therefore hope that our readers will discover a certain unity of thinking among different domains of physics. In this case, this book will not have been written in vain.

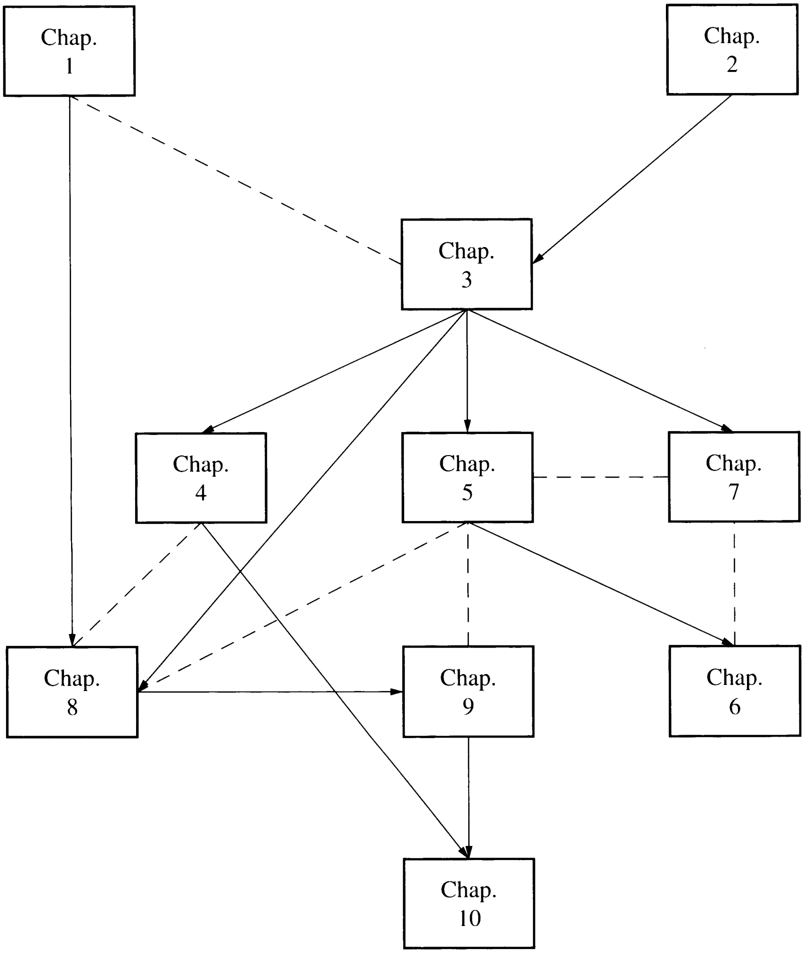
This book was written at the instigation of the Troisième Cycle de la Physique en Suisse Romande, and in particular of J.-J. Loeffel. We are grateful to the many colleagues who provided useful suggestions or enlightenment on various points, namely to the late P. Huguenin and to B. Jancovici, D. Pavuna, J.-P. Perroud and G. Wanders. V. Savona helped with the elaboration of the exercises at the end of each chapter. We thank R. Fernandez for encouraging us to translate the book. S. Goldfarb translated and typed the whole text, including the numerous equations; D. Watson helped us to formulate additional material; L. Klinger and L. Trento drew the figures; our thanks go to all these people for their contributions. Finally we are indebted to the Institute of Condensed-Matter Physics and the Physics Section of the University of Lausanne, as well as to the Physics Department of the Swiss Federal Institute of Technology, Lausanne, for financial support.

August 2001

*Philippe A. Martin*
*François Rothen*

# Table of Contents

# Reader's Guide




Logical organization of the material: the arrows indicate a recommended reading order

Chapter 1 offers an incomplete summary of quantum mechanics and of classical field theory. By consequence, it recalls notions which are useful, but which are generally part of an undergraduate repertoire. It is left to the reader to determine the necessity for a thorough study of its contents.