

Texts and Monographs in Physics

Series Editors: R. Balian W. Beiglböck H. Grosse E. H. Lieb
N. Reshetikhin H. Spohn W. Thirring

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Sergei V. Ketov

Quantum Non-linear Sigma-Models

From Quantum Field Theory
to Supersymmetry,
Conformal Field Theory,
Black Holes and Strings

With 51 Figures



Springer

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To Tatiana, Michael and Denise

Preface

The idea for this book came to me after visiting DESY and CERN in 1996 and 1997. For a long time, two-dimensional Non-Linear Sigma-Models (NLSM) served as a useful laboratory for the study of perturbative and non-perturbative properties of four-dimensional non-Abelian gauge theories, since they share many remarkable features like renormalizability, asymptotic freedom, solitons, confinement, etc. [1]. For instance, the low-energy effective physics of pions in four-dimensional Quantum Chromodynamics (QCD) receives the most natural description in terms of the principal NLSM whose solitonic solutions (skyrmions) can be interpreted as baryons [2]. In fact, NLSM are also important for spontaneous symmetry breaking, extended supersymmetry and supergravity, conformal field theory, gravity and string theory. This book is entirely devoted to recent applications of NLSM in various dimensions.

In the late 1980s and while in Russia, I wrote the book [3] entitled ‘Non-linear Sigma-Models in Quantum Field Theory and Strings’, which was eventually published in Russian by the Nauka Publishers in 1992. This book is *not* a translation of my earlier book into English, though it shares about one third of its content with the Russian edition. The main additions include the two-dimensional Wess-Zumino-Novikov-Witten (WZNW) models in conformal field theory and strings, gauging NLSM isometries, four-dimensional NLSM with $N = 2$ extended supersymmetry in the context of Seiberg-Witten theory and M-theory, $N = 2$ strings and D-brane dynamics.

This book is *not* a collection of all known facts about NLSM. For instance, any extended discussion of NLSM solitonic solutions and their low-energy scattering, as well as many standard applications of NLSM in condensed matter and low-energy physics of hadrons, were intentionally excluded, since there is already extensive monographic literature on these, see e.g., [4, 5, 6, 7, 8] and references therein. The book is not merely aimed at providing the formal mathematical background to the field theory of NLSM and their quantization. In fact, I have adopted a more ‘applied’ approach for my presentation that is oriented towards practitioners in quantum field theory, supersymmetry, gravity and modern string theory. The book content is heavily based on my original papers, so that the selected material considerably reflects my own research interests in the past. Nevertheless, this book is not just a col-

lection of my papers, and it does not duplicate any existing review. Although extensive, the list of literature at the end of the book cannot be considered an exhaustive bibliography on NLSM by any means. I would like to apologize to those authors whose contributions escaped my attention or were not mentioned.

This book cannot replace an introduction to quantum field theory, conformal field theory or string theory, though I have done my best to make it readable for those who are merely familiar with the foundations of quantum field theory and classical general relativity, and who are interested in various applications of the NLSM techniques. Therefore, this book should certainly be accessible to Ph.D. students wishing to do research in either quantum field theory, strings, supersymmetry, conformal field theory or related areas of mathematical physics, as well as to those readers interested in phenomenologically oriented applications of the formalism presented here.

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Hannover, July 2000

Sergei V. Ketov

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