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F. Thomasset

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Edited by D. Book

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A. F. Nikiforov, S. K. Suslov, V. B. Uvarov

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W. D. D'haeseleer, W. N. G. Hitchon, J. D. Callen, J. L. Shohet

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K. K. Sabelfeld

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M. L. Wilkins

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ISSN 1434-8322
ISBN 978-3-642-08315-0

Library of Congress Cataloging-in-Publication Data

Wilkins, Mark L., 1922- Computer simulation of dynamic phenomena / by Mark L. Wilkins. p. cm. - (Scientific computation) Includes bibliographical references.
ISBN 978-3-642-08315-0 ISBN 978-3-662-03885-7 (eBook)
DOI 10.1007/978-3-662-03885-7
1. Hydrodynamics-Computer simulation. 2. Gas dynamics-Computer simulation.
3. Elastoplasticity-Computer simulation. I. Title. II. Series. QC151.W495 1999 532'.5-ddc21
99-42088 CIP

Originally published by Springer-Verlag Berlin Heidelberg New York in 1999

Softcover reprint of the hardcover 1st edition 1999

Cover design: *design & production* GmbH, Heidelberg
Typesetting with LaTeX: PTP - Berlin, Stefan Sossna
SPIN 10554734 55/3144/di - 5 4 3 2 1 0 - Printed on acid-free paper

Preface

This text describes computer programs for simulating phenomena in hydrodynamics, gas dynamics, and elastic plastic flow in one, two, and three dimensions. Included in the two-dimensional program are Maxwell's equations and thermal and radiation diffusion. The programs were developed by the author during the years 1952–1985 at the Lawrence Livermore National Laboratory.

The largest main-frame computers available in the early 1950s were required to solve hydrodynamic problems in one space dimension by using forty mass points. Subsequently, numerical methods were developed for solving problems in two and three space dimensions, but application of these methods had to wait until the main-frame computers were large enough to tackle meaningful problems. At the present time, lap-top computers can use these methods to solve problems in three space dimensions with the detail of 10 000 mass points.

The numerical procedures described in the text permit the exact conservation of physical properties in the solutions of the fundamental laws of mechanics: (1) conservation of mass, (2) conservation of momentum, (3) conservation of energy. The laws of mechanics are universal in their application. Examples are given for the same computer simulation programs solving problems of penetration mechanics, surface waves from earthquakes, shock waves in solids and gases, failure of materials.

Many important concepts in mathematics have two or more equivalent definitions. A particularity of this book resides in the choice of the physical solid rather than the mathematical point as the basic concept in the schematization of the material system. The gradient of a scalar point function is expressed in the form of a surface integral. The divergence and curl of vector point functions are represented in a similar manner. The integral definition of a derivative defines a control volume in a scalar or vector field. The difference in the flux in and out of the control volume is the induced change in the quantities in the control volume. Thus, the numerical system itself is in a conservation form with zero truncation error in the solution of the partial differential equations. Lagrange coordinates are employed, which permits the history of the behavior of a mass particle to be followed. The mass particle with its corresponding control volume can be made arbitrarily small. Proceeding with the mathematics in the form of a finite material system is

consistent with the observation that the components of a vector force can only be measured on a surface.

The approach here, of describing the mathematics in terms of mechanics instead of the reverse, can lead to slightly different results. For example, some of the sacred tenets of plasticity theory, which is a mathematical theory, are not necessary when a physics approach is applied to the problem.

The closure to the solutions of the three fundamental laws of mechanics is the model for the material behavior. When the model or material equation of state is known, the engineer can use the simulation program to predict the consequences of a material structure to a given situation. The programs are especially useful in studying structures submitted to failure conditions that would be very expensive to investigate by experiment.

With the ability to solve the three laws of mechanics to any degree of accuracy, the problem can be turned around for the materials scientist to test a hypothesis. The simulation of an experiment that measures an observable, which by itself may yield no information of single significance, can be used to construct a model of material behavior. A mechanical equation of state can be developed in this manner. If the theoretical model of material behavior based on fundamental assumptions cannot produce the same result, the theory must be revised.

In my work I have relied on the skills of many colleagues at the Lawrence Livermore National Laboratory. D.E. Giroux assisted by T. Suyehiro developed the programming for the two-dimensional simulation program HEMP[1]. The program was further developed to include multiple sliding by Alan Leibee and Karen Warren. S.J. French programmed early versions of the three-dimensional program HEMP 3D in the late 1960s.

Eugene Cronshagen developed the production version of HEMP 3D with vector programming, grid generators, and 3D graphics. Robert Gulliford and David Turner programmed the sliding interfaces and fracture models in the HEMP 3D program. Robert Dickens programmed the magnetohydrodynamic version of HEMP.

Material models play an essential role in the usefulness of a computer simulation program. Here I would like to acknowledge the many contributions made by Jack Reaugh and Michael Guinan.

Livermore,
December 1998 *Mark L. Wilkins*

[1] HEMP is an acronym stemming from the words Hydrodymanic, Elastic, Magneto, and Plastic.

Table of Contents

Notation

ρ	local density
ρ_0	reference density
V	relative volume $= \rho_0/\rho$
$\boldsymbol{W}$	velocity vector
$\dot{X}, \dot{Y}$	components of velocity vector
Σ	stress tensor
t	time
X, Y	space coordinate with cylindrical symmetry about the X axis
θ	direction perpendicular to the X-Y plane
$\boldsymbol{H}$	magnetic field
$\mu_{\mathrm{m}}\Phi$	magnetic flux
μ_{m}	magnetic permeability
P	material pressure + radiation pressure
q	artificial viscosity
E	internal energy (per original volume)$= c_{\mathrm{v}}T + a_{\mathrm{R}}VT^4$
ε_{m}	material internal energy (per original volume) $= c_{\mathrm{v}}T$
$\mathrm{d}Z$	change in intemal energy by distortion and magnetic pressure stresses
S	surface
$\mathrm{d}s$	element of surface
$\mathbf{n}$	outward normal to surface
$\mathrm{d}\mathbf{l}$	line element of contour (C)
A	zone area
M	zone mass
$\dot{\varepsilon}_{XY}, \dot{\varepsilon}_{XX}, \dot{\varepsilon}_{YY}, \dot{\varepsilon}_{\theta\theta}$	components of the strain rate tensor
$T_{XY}, s_{XX}, s_{YY}, s_{\theta\theta}$	components of the deviatric stress tensor

T	temperature
K	thermal conductivity
λ	Rosseland mean free path
Λ	$K + 4/3 a_{\mathrm{R}} c \lambda T^3$ = transmissivity
$\boldsymbol{E}$	electric field
$\boldsymbol{J}$	current density
C	electrical conductivity
c_{v}	specific heat
a_{R}	Stefan-Boltzmann constant
c	velocity of light

Units

Symbol	Quantity	Units	SI equivalent
eu	energy unit	10^{12} ergs	100 kJ
ρ	density	$\mathrm{gm/cm^3}$	$10^3\ \mathrm{kg/m^3}$
V	relative volume	dimensionless	—
E	internal energy per original volume	$\frac{\mathrm{eu}}{\mathrm{gm}} \times \rho_0 = \frac{\mathrm{eu}}{\mathrm{cm^3}}$	$\frac{100\ \mathrm{GJ}}{\mathrm{m^3}}$
t	time	$\mu\mathrm{s} = 10^{-6}$ s	μs
X, Y	space coordinates	cm	10 mm
$\dot{X}, \dot{Y}$	velocity	$\mathrm{cm/\mu s}$	10 km/s
T	temperature	K	K
P	pressure (and stress)	Mbar $= 10^{12}$ dynes/cm^2	100 GPa
$\boldsymbol{H}$	magnetic field	10^6 gauss	100 tesla
$\boldsymbol{E}$	electrical field	10^4 volts/cm	MV/m
$\boldsymbol{J}$	current density	10^7 amps/cm^2	100 GA/m^2
C	electrical conductivity	10^3 mho/cm	100 kmho/m
λ	Rosseland mean free path	cm	10 mm
K	coefficient of thermal conduction	$\frac{\mathrm{eu}}{(\mathrm{cm})(\mathrm{K})(\mu\mathrm{s})}$	$10\ \mathrm{TJ/m \cdot K \cdot s}$
c_v	specific heat at constant volume	$\frac{\mathrm{eu} \times \rho_0}{(\mathrm{gm})(\mathrm{K})} = \frac{\mathrm{eu}}{(\mathrm{K})(\mathrm{cm^3})}$	$100\ \mathrm{GJ/K \cdot m^3}$
a_R	Stefan–Boltzmann radiation constant	$\frac{7.56 \times 10^{-27}\ \mathrm{eu}}{(\mathrm{cm^3})(\mathrm{K^4})}$	$756\ \mathrm{aJ/K^4 \cdot m^3}$
c	velocity of light	$3 \times 10^4\ \mathrm{cm/\mu s}$	3×10^8 m/s

Symbol	Quantity	Units	SI equivalent
μ_m	magnetic permeability	1	1

T = tera = 10^{12}; G = giga = 10^9; M = mega = 10^6
k = kilo = 10^3; a = atto = 10^{-18}