

# Texts and Monographs in Physics

---

*Series Editors:* R. Balian W. Beiglböck H. Grosse E. H. Lieb  
N. Reshetikhin H. Spohn W. Thirring

Springer-Verlag Berlin Heidelberg GmbH

# Texts and Monographs in Physics

---

*Series Editors:* R. Balian W. Beiglböck H. Grosse E. H. Lieb  
N. Reshetikhin H. Spohn W. Thirring

**From Microphysics to Macrophysics  
I + II** Methods and Applications  
of Statistical Physics By R. Balian

**Multi-Hamiltonian Theory  
of Dynamical Systems**  
By M. Błaszak

**Quantum Mechanics:  
Foundations and Applications**  
3rd enlarged edition By A. Böhm

**Operator Algebras and Quantum  
Statistical Mechanics I + II** 2nd edition  
By O. Bratteli and D. W. Robinson

**Statistical Methods in Quantum  
Optics 1.** Master Equations and  
Fokker–Planck Equations  
By H. J. Carmichael

**Geometry of the Standard Model  
of Elementary Particles**  
By A. Derdzinski

**Scattering Theory of Classical  
and Quantum  $N$ -Particle Systems**  
By J. Dereziński and C. Gérard

**Effective Lagrangians  
for the Standard Model**  
By A. Dobado, A. Gómez-Nicola,  
A. L. Maroto and J. R. Peláez

**Quantum**  
The Quantum Theory of Particles, Fields,  
and Cosmology By E. Elbaz

**Quantum Relativity**  
A Synthesis of the Ideas of Einstein  
and Heisenberg  
By D. R. Finkelstein

**Quantum Mechanics I + II**  
By A. Galindo and P. Pascual

**The Elements of Mechanics**  
By G. Gallavotti

**Local Quantum Physics**  
Fields, Particles, Algebras  
2nd revised and enlarged edition  
By R. Haag

**Supersymmetric Methods in Quantum  
and Statistical Physics** By G. Junker

**$CP$  Violation Without Strangeness**  
Electric Dipole Moments of Particles,  
Atoms, and Molecules  
By I. B. Khriplovich and S. K. Lamoreaux

**Quantum Groups  
and Their Representations**  
By A. Klimyk and K. Schmüdgen

**Statistical Mechanics of Lattice Sys-  
tems 1.** Closed-Form and Exact Solutions  
2nd enlarged and revised edition  
By D. A. Lavis and G. M. Bell

**Statistical Mechanics of Lattice Sys-  
tems 2.** Exact, Series and Renormaliza-  
tion Group Methods  
By D. A. Lavis and G. M. Bell

**Fields, Symmetries, and Quarks** 2nd  
revised and enlarged edition By U. Mosel

**Quantum Entropy and Its Use**  
By M. Ohya and D. Petz

**Generalized Coherent States  
and Their Applications**  
By A. Perelomov

**Path Integral Approach  
to Quantum Physics** An Introduction  
2nd printing By G. Roepstorff

**Renormalization** An Introduction  
By M. Salmhofer

**Finite Quantum Electrodynamics**  
The Causal Approach 2nd edition  
By G. Scharf

**From Electrostatics to Optics**  
A Concise Electrodynamics Course  
By G. Scharf

**The Mechanics and Thermodynamics  
of Continuous Media** By M. Šilhavý

**Large Scale Dynamics of Interacting  
Particles** By H. Spohn

**The Theory of Quark and Gluon  
Interactions** 2nd completely revised  
and enlarged edition By F. J. Ynduráin

**Relativistic Quantum Mechanics  
and Introduction to Field Theory**  
By F. J. Ynduráin

---

Ulrich Mosel

---

# Fields, Symmetries, and Quarks

Second, Revised and Enlarged Edition  
With 30 Figures



Springer

Professor Dr. Ulrich Mosel  
Institut für Theoretische Physik  
Justus-Liebig-Universität Giessen  
Heinrich-Buff-Ring 16  
D-35392 Giessen, Germany

### *Editors*

---

Roger Balian  
CEA  
Service de Physique Théorique de Saclay  
F-91191 Gif-sur-Yvette, France

Wolf Beiglböck  
Institut für Angewandte Mathematik  
Universität Heidelberg  
Im Neuenheimer Feld 294  
D-69120 Heidelberg, Germany

Harald Grosse  
Institut für Theoretische Physik  
Universität Wien  
Boltzmannngasse 5  
A-1090 Wien, Austria

Elliott H. Lieb  
Jadwin Hall  
Princeton University, P.O. Box 708  
Princeton, NJ 08544-0708, USA

Nicolai Reshetikhin  
Department of Mathematics  
University of California  
Berkeley, CA 94720-3840, USA

Herbert Spohn  
Zentrum Mathematik  
Technische Universität München  
D-80290 München, Germany

Walter Thirring  
Institut für Theoretische Physik  
Universität Wien  
Boltzmannngasse 5  
A-1090 Wien, Austria

---

The 1st edition was published under the title: *Fields, Symmetries, and Quarks*  
© 1989 McGraw-Hill Hamburg

---

ISSN 0172-5998  
ISBN 978-3-642-08458-4

Library of Congress Cataloging-in-Publication Data applied for.

Die Deutsche Bibliothek - CIP-Einheitsaufnahme

**Mosel, Ulrich:** *Fields, symmetries, and quarks* / Ulrich Mosel. - 2., rev. and enl. ed.

ISBN 978-3-642-08458-4 ISBN 978-3-662-03841-3 (eBook)

DOI 10.1007/978-3-662-03841-3

This work is subject to copyright. All rights are reserved, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, reuse of illustrations, recitation, broadcasting, reproduction on microfilm or in any other way, and storage in data banks. Duplication of this publication or parts thereof is permitted only under the provisions of the German Copyright Law of September 9, 1965, in its current version, and permission for use must always be obtained from Springer-Verlag. Violations are liable for prosecution under the German Copyright Law.

© Springer-Verlag Berlin Heidelberg 1999

Originally published by Springer-Verlag Berlin Heidelberg New York in 1999

Softcover reprint of the hardcover 2nd edition 1999

The use of general descriptive names, registered names, trademarks, etc. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant protective laws and regulations and therefore free for general use.

Typesetting: Camera ready by the author using a Springer  $\text{\TeX}$  macro package

Cover design: *design & production* GmbH, Heidelberg

Computer to plate: Mercedesdruck, Berlin

SPIN: 10652003 55/3144/di - 5 4 3 2 1 0 - Printed on acid-free paper

# Preface

This revised and extended edition of the book *Fields, Symmetries, and Quarks*, originally published by McGraw-Hill Book Company, Hamburg, 1989, contains a new chapter on electroweak interactions which has also grown out of lectures that I have given in the meantime. In addition, a number of changes, mainly in the metric used, in the discussion of the theory of strong interactions, QCD, and in the chapter on hadron physics, have been made and errors have been corrected.

The motivation for this book, however, is still the same as it was 10 years ago: This is a book on quantum field theory and our present understanding of leptons and hadrons for advanced students and the non-specialists and, in particular, the experimentalists working on problems of nuclear and hadron physics.

I am grateful to Dr. S. Leupold for a very careful reading of the revised manuscript, many corrections, and helpful suggestions and to C. Traxler for producing the figures and for constructive discussions.

Giessen, September 1998

*Ulrich Mosel*

# Preface to the First Edition

This is a book on the elements of quantum field theory, symmetries, gauge field theories and phenomenological descriptions of hadrons with special emphasis on topics relevant to nuclear physics. It has been written by a nuclear theorist, in particular for nuclear physicists, including experimentalists, and in general for scientists who need a working knowledge of field theory, symmetry principles of elementary particles and their interactions and the quark substructure of hadrons without having to be experts in each of these fields.

At the highest energies now employed in nuclear physics we start to probe phenomena of high-energy and particle physics in the nuclear environment. It is one of the purposes of this book to provide the necessary theoretical background for this study of fundamental symmetries in nuclear processes, subnuclear degrees of freedom, multi-quark systems and their interactions in a coherent, detailed, but nevertheless didactic way that stresses physical principles and introduces theoretical formalisms only as far as they are really necessary for a systematic understanding of the material discussed.

This book can serve a second purpose: it has grown out of notes for lectures that I gave at the University of Giessen first in 1984 and then again in 1987. The lectures were attended by students in their sixth semester (third year) who had a background in nonrelativistic and relativistic quantum theory, including the method of second quantization and the Klein-Gordon and Dirac equations, but no knowledge of field theory. Since field theory is usually not a mandatory course for our physics students, many of them leave the university without knowing much about it. Most of them have never heard about gauge field theories and their importance for our present day understanding of basic interactions. It was the purpose of this course to provide the essential ideas of these fields for students who were interested in a deeper understanding of the forces of nature but did not necessarily want to become specialists in field theory or particle physics. A course based on this book, perhaps shortening or leaving out the last two more specialized chapters on bag models, can fulfill their needs.

The book starts out with an elementary introduction into classical field theory, the quantization of free fields and a very short treatment of interacting fields in its first three chapters. Gauge field theories require a working knowl-

edge of global symmetries in field theories. This topic is therefore discussed in detail in the second part of this book.

The third part is then concerned with the general structure of gauge field theories and contains a thorough discussion of the still less widely known features of non-Abelian gauge field theories. The two examples of most importance to nuclear physicists are clearly the theory of electroweak interactions, which unifies electromagnetic and weak interactions, and Quantum Chromodynamics (QCD), which describes the strong interactions of hadrons. Because there are already many excellent textbook treatments of the electroweak interactions available, I have chosen here to discuss only QCD because of its importance for our understanding of hadronic matter. QCD predicts bag-like ensembles of quarks that make up the nucleons; it also predicts the existence of a new phase of nuclear matter, the so-called Quark–Gluon Plasma. It is here that nuclear physics and particle physics meet.

The last two chapters then contain detailed discussions of phenomenological bag models. The MIT bag is discussed in much detail so that all theoretical calculations can be followed step by step. Since in all other bag-models the calculational methods and steps are essentially identical this chapter should enable the reader to actually perform such calculations unaided. A last chapter finally discusses the topological bag models which have become quite popular over the last few years. All of these discussions have not until now been available in the textbook literature.

The bibliography contained in this book will enable readers to study many topics in more detail and will help them to go on to topics not covered here. I have in general tried, whenever available, to cite only recent review articles about the topics treated. There the detailed references to the original literature can be found.

I am grateful to many of my students for helpful comments and suggestions on early versions of the lecture notes. I am particularly thankful to one of my graduate students, Bernhard Blättel, who worked through the whole book correcting numerous mistakes but also suggesting improvements in language, style and presentation. He also reminded me constantly of the available background knowledge of students. I am also very much indebted to Elke Szauter for typing the whole manuscript and making numerous alterations; finally I would like to thank Dieter Masak and Ulrich Post for their help in preparing the figures for this book.

# Contents

---

## Part I. Preliminaries

---

<b>1. Units and Metric</b> .....	3
1.1 Units .....	3
1.2 Metric and Notation .....	4

---

## Part II. Fundamentals of Field Theory

---

<b>2. Classical Fields</b> .....	9
2.1 Equations of Motion .....	9
2.1.1 Examples .....	11
2.2 Symmetries and Conservation Laws .....	14
2.2.1 Geometrical Space–Time Symmetries .....	15
2.2.2 Internal Symmetries .....	17
<b>3. Free Fields and Their Quantization</b> .....	21
3.1 Classification of Fields .....	21
3.2 Scalar Fields .....	23
3.2.1 Quantization of the Hermitian Scalar Field .....	24
3.2.2 Quantization of the Charged Scalar Field .....	27
3.3 Vector Fields .....	29
3.3.1 Massive Vector Fields .....	29
3.3.2 Massless Vector Fields .....	30
3.4 Fermion Fields .....	32
3.4.1 Dirac Equation .....	32
3.4.2 Lagrangian for Fermion Fields .....	34
3.4.3 Quantization of the Dirac Field .....	35
3.4.4 Massless Fermions .....	38
3.4.5 Neutrinos .....	40
3.5 Transition Rates in Quantum Field Theory .....	44
3.6 Quantum Mechanical Consistency .....	52



---

**Part III. Global Symmetries**


---

<b>4. Symmetries of Meson and Baryon Systems</b> .....	57
4.1 $U(1)$ Symmetry .....	57
4.1.1 Properties of the Group $U(1)$ .....	58
4.1.2 Structure of the Nucleon Lagrangian .....	59
4.2 $SU(2)$ Symmetry .....	60
4.2.1 Properties of the Group $SU(2)$ .....	60
4.2.2 General Definitions .....	64
4.2.3 Application to the Pion–Nucleon System .....	64
4.2.4 Structure of $SU(2)$ Multiplets .....	72
4.3 $SU(3)$ Symmetry .....	74
4.3.1 Properties of the Group $SU(3)$ .....	75
4.3.2 Structure of $SU(3)$ Multiplets .....	78
4.3.3 Assignments of Hadrons to $SU(3)$ Multiplets .....	83
4.3.4 $SU(3)$ Symmetry Breaking .....	85
<b>5. Quarks</b> .....	87
5.1 Construction of $SU(3)$ Multiplets .....	87
5.1.1 Construction of the Representation $3 \otimes \bar{3}$ .....	89
5.1.2 Construction of the Representation $3 \otimes 3 \otimes 3$ .....	90
5.2 State Vectors for the Multiplets .....	92
5.2.1 Tensor Algebra .....	93
5.2.2 Hadron Multiplets .....	96
5.3 Color Degree of Freedom .....	100
<b>6. Chiral Symmetry</b> .....	103
6.1 Phenomenology of $\beta$ -Decay .....	103
6.1.1 Leptonic $\beta$ -Decay .....	103
6.1.2 Semileptonic $\beta$ -Decay .....	104
6.2 Current Conservation in Strong Interactions .....	106
6.2.1 Vector Current Conservation .....	106
6.2.2 Axial Vector Current Conservation .....	108
6.3 Chiral Symmetry Group .....	110
6.3.1 Chiral Symmetry Transformations for the Fermions ...	110
6.3.2 Chiral Symmetry Transformations for the Mesons ...	112
<b>7. Spontaneous Global Symmetry Breaking</b> .....	115
7.1 Goldstone Theorem .....	115
7.1.1 Goldstone Bosons .....	116
7.2 Examples of the Goldstone Mechanism .....	119
7.2.1 Spontaneous Breaking of a Global Non-Abelian Symmetry .....	119
7.2.2 $\sigma$ -Model .....	121
7.2.3 Nambu–Jona–Lasinio Model .....	129

---

**Part IV. Local Gauge Symmetries**

---

**8. Gauge Field Theories** ..... 135

8.1 Conserved Currents in QED ..... 135

8.2 Local Abelian Gauge Invariance ..... 137

8.3 Non-Abelian Gauge Fields ..... 139

8.3.1 Lagrangian for Non-Abelian Gauge Field Theories .... 139

8.3.2 Properties of Non-Abelian Gauge Field Theories ..... 144

**9. Spontaneous Symmetry Breaking in Gauge Field Theories** 147

9.1 Higgs Mechanism ..... 147

9.2 Spontaneous Breaking of a Local Non-Abelian Symmetry .... 150

9.3 Summary of the Higgs Mechanism ..... 155

---

**Part V. Electroweak Interaction**

---

**10. Weak Interactions of Quarks and Leptons** ..... 159

10.1 Phenomenological Introduction ..... 159

10.1.1 Strangeness Changing Weak Decays ..... 159

10.1.2 Neutral Currents ..... 160

10.2 Intermediate Vector Bosons ..... 161

10.3 Fundamentals of a Theory of Weak Interactions ..... 164

**11. Electroweak Interactions of Leptons** ..... 167

11.1 Leptonic Multiplets and Interactions ..... 167

11.1.1 Electroweak Currents ..... 174

11.2 Lepton Masses ..... 175

11.3 Electroweak Interactions ..... 176

11.3.1 Generalization to Other Leptons ..... 180

11.4 Parameters of the Lagrangian ..... 180

11.4.1 Charged Current Experiments ..... 180

11.4.2 Neutral Current Experiments ..... 182

**12. Electroweak Interactions of Quarks** ..... 187

12.1 Hadronic Multiplets ..... 187

12.1.1 Hadron Masses ..... 190

**13. Electroweak Interactions of Quarks and Leptons** ..... 193

13.1 Lagrangian of Electroweak Interactions ..... 193

13.2 Standard Model ..... 194

<b>14. CP Invariance of Electroweak Interactions</b> .....	197
14.1 Kobayashi–Maskawa Matrix .....	197
14.2 Unitarity of the KM Matrix .....	198
14.3 $K^0$ Decay and CP Violation .....	201
14.4 CP Invariance and the KM Matrix.....	203

---

**Part VI. Strong Interaction**

---

<b>15. Quantum Chromodynamics</b> .....	209
15.1 Gauge Group for Strong Interactions .....	209
15.2 QCD Lagrangian .....	211
15.3 Properties of QCD.....	213
15.3.1 Scale Invariance .....	213
15.3.2 Chiral Invariance .....	214
15.3.3 Antishielding and Confinement .....	215
15.3.4 Deconfinement Phase Transition.....	217

---

**Part VII. Hadron Structure**

---

<b>16. Bag Models of Hadrons</b> .....	223
16.1 Potential Well in the Dirac Theory .....	223
16.2 The MIT Bag .....	228
16.2.1 Fermions in the MIT Bag .....	229
16.2.2 Hadron Masses .....	232
16.2.3 Gluons in the MIT Bag .....	234
16.2.4 Hyperfine Structure of Bag States .....	235
16.2.5 Magnetic Moments of the Nucleon .....	243
16.2.6 Axial Vector Current .....	246
16.2.7 Chiral Symmetry in the MIT Bag .....	251
<b>17. Soliton Models of Hadrons</b> .....	257
17.1 Skyrmion Model.....	258
17.2 Hybrid Chiral Bag Model.....	262
17.3 Linear $\sigma$ -Model .....	269
17.4 Friedberg–Lee Soliton Bag Model.....	272
17.5 NJL Soliton Model .....	274

---

**Part VIII. Appendices**

---

<b>A. Solutions of the Free Dirac Equation</b> .....	279
A.1 Properties of Free Dirac States .....	279
A.2 Massless Fermions .....	284
A.3 Dirac and Majorana Fields .....	286
<b>B. Explicit Quark States for Hadrons</b> .....	289
<b>C. Table of Hadron Properties</b> .....	293
<b>Bibliography by Subject</b> .....	297
<b>References</b> .....	303
<b>Index</b> .....	307