

# Elementary Particles and Their Interactions

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Quang Ho-Kim Xuan-Yem Pham

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# Elementary Particles and Their Interactions

Concepts and Phenomena

With 116 Figures, 36 Tables, Numerous Examples,  
and 102 Problems with Selected Solutions



Springer

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*To our families*

# Preface

The last few decades have seen major advances in the physics of elementary particles. New generations of particle accelerators and detectors have come into operation, and have successfully contributed to improving the quantity and quality of data on diverse interaction processes and to the discoveries of whole new families of particles. At the same time, important new ideas have emerged in quantum field theory, culminating in the developments of theories for the weak and strong interactions to complement quantum electrodynamics, the theory of the electromagnetic force. The simplest of the new theories that are at the same time mathematically consistent and physically successful constitute what is known as the standard model of the fundamental interactions. This book is an attempt to present these remarkable advances at an elementary level, making them accessible to students familiar with quantum mechanics, special relativity, and classical electrodynamics.

The main content of the book is roughly divided into two parts; one on theories to lay the foundation and the other on further developments of concepts and descriptions of phenomena to prepare the student for more advanced work. After a brief overview of the subject and a presentation of some basic ideas, two chapters which deal mostly with relativistic one-body wave equations, quantization of fields, and Lorentz invariance follow. In the spirit of the practical approach taken in this book, a heuristic derivation of the Feynman rules is given in the fourth chapter, where the student is shown how to calculate cross-sections and decay rates at the lowest order. The following chapter contains a discussion on discrete symmetries and the concept of symmetry breaking. Isospin is introduced next as the simplest example of internal symmetries in order to ease the reader into the notion of unitary groups in general and of  $SU(3)$  in particular, which is discussed next together with the recent discoveries of new particles. The next two chapters present the standard model of the fundamental interactions. We make contact with experiments in subsequent chapters with detailed studies of some fundamental electroweak processes, such as the deep inelastic lepton–nucleon scattering, the CP violation in the neutral K mesons, the neutrino oscillations and the related problem of the solar neutrino deficit, and finally, the  $\tau$  lepton decay, which touch upon many aspects of weak interactions. The very high precision of the data that is now attained in some of these processes requires a careful examination of higher-order effects. This leads to a detailed

study of one-loop QCD corrections to weak interactions. The next chapter demonstrates the remarkable property of asymptotic freedom of quantum chromodynamics and introduces the powerful concept of the renormalization group which plays a central role in many phenomena. The heavy flavors of quarks, which pose new questions on several aspects of interactions and could open windows on the ‘new’ physics, form the subject of a separate chapter. We close with a review of the present status of the standard model and, briefly, of its extensions. Selected solutions to problems are given. Finally, important formulas are collected in an Appendix for convenient reference.

In writing this book we have constantly borne in mind the beginning student learning the subject for the first time. For this reason we have avoided a presentation of the formalism based either on canonical quantization or path integral methods. We have adopted instead a decidedly more practical approach based on perturbative field theory. Many particle phenomena may thus be described in detail early in the book, and the student, in turn, can carry out actual calculations. The importance of the physical point of view is further emphasized by the many examples found throughout the book. The first part of the book gives the student the basic (and some extra) material needed to follow the arguments leading to the standard model and to understand the physics that flows from it. The second part is an attempt to reflect recent advances in experimental particle physics (such as neutrino oscillations, B meson physics, and precision tests of electroweak processes). These topics are selected mainly on the strength of their lasting intrinsic value or because they bring out some novel physics. Whatever the motivations, we introduce all topics at an elementary level, work out the calculations in detail, and carry the development to the point where the reader can start deepening his or her own understanding through a meaningful independent study.

We owe thanks to our teachers, students, and colleagues for the physics they have taught us. Many have helped us in our present project. We are in particular grateful to Pierre Fayet, Michel Gourdin, Chi-Sing Lam, Serguey Petcov, and Pham Tri-Nang for reading parts of the book and for making judicious comments and suggestions. Thanks are also due to Dr. Hans Kölsch, our editor at Springer for a pleasant and fruitful collaboration. One of us (QHK) acknowledges with gratitude the financial support given by the Natural Sciences and Engineering Research Council of Canada and the gracious hospitality extended to him by the Laboratoire de Physique Théorique et Hautes Énergies (Université Paris VI et Université Paris VII) and the Laboratoire de Physique Théorique et Modélisation (Université Cergy-Pontoise). Finally, we are greatly indebted to our families, to whom this work is dedicated, for their support and encouragement throughout the writing of this book.

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