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A Series of Comprehensive Studies in Mathematics

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Sheaves on Manifolds

With a Short History
«Les débuts de la théorie des faisceaux»
By Christian Houzel

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Preface

For a long time after its introduction by Leray, sheaf theory was mainly applied to the theory of functions of several complex variables or to algebraic geometry, until it became a basic tool for almost all mathematicians, and cohomology a natural language for many people.

However, while there exists an extensive literature dealing with cohomology of sheaves (e.g. the famous book by Godement) or even with derived functors, there are in fact very few books developing sheaf theory within the beautiful framework of derived categories although its necessity is becoming more and more evident. Most of the constructions of the theory take on their full strength in this context, or even, do not make sense outside of it. This is particularly evident for the Poincaré-Verdier duality, which appeared in the sixties, as well as for the Sato microlocalization, introduced in 1969, which is only beginning to be fully understood.

Since the seventies, other fundamental ideas have emerged and sheaf theory (on manifolds) naturally includes the "microlocal" point of view. Our aim is to present here a self-contained work, starting from the beginning (derived categories and sheaves), dealing in detail with the main features of the theory, such as duality, Fourier transformation, specialization and microlocalization, micro-support and contact transformations, and also to give two main applications. The first of these deals with real analytic geometry, and includes the concepts of constructible sheaves, subanalytic cycles, Euler-Poincaré indices, Lefschetz formula, perverse sheaves, etc. The second one is the theory of linear partial differential equations, including D-modules, microfunctions, elliptic and micro-hyperbolic systems, and complex quantized contact transformations.

With this book we hope to illustrate the deep links that tie together branches of mathematics at first glance seemingly disconnected, such as for example here, algebraic topology and linear partial differential equations. At the same time, we want to emphasize the essentially geometrical nature of the problems encountered (most obvious in the involutivity theorem for sheaves), and to show how efficient the algebraic tools introduced by Grothendieck are in solving them, even for an analyst.

Of course, many important applications of the theory are just touched upon, such as for instance the theory of microdifferential systems (complete monographs on the topic are however available now), others are simply omitted, such as representation theory and equivariant sheaf theory.

Finally, we want to express our thanks to C. Houzel who agreed to write a short history of sheaf theory, to L. Illusie who helped us when preparing the "Historical Notes", to those who went through various parts of the book and made constructive comments, especially E. Andronikof, A. Arabia, J-M. Delort, E. Leichtnam and J-P. Schneiders, and also to Catherine Simon at Paris-Nord University and the secretarial staff of the RIMS at Kyoto, who had the patience to type the manuscripts.

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