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Finite Quantum Electrodynamics

With 4 Figures

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Preface

Why a book on quantum electrodynamics? Why not a book on electro-weak interactions or even one including quantum chromodynamics, when everybody knows today that all these theories are essentially of the same nature and the trend is towards unification? Our restriction has a reason.

A course on quantum field theory usually starts with classical field theory. Soon the quantization of free fields is discussed. The professor feels happy and the students follow easily. Then comes the moment when interactions are introduced. If the professor is honest, he then says that he cannot tell what the precise meaning of the interacting fields is. He adds that this question is irrelevant for what follows, because the formulae on the blackboard only serve the purpose of arriving at the perturbation theory. The theory is, so to say, *defined* perturbatively. He still feels well, the students, however, much less so. The point then arrives when the first non-trivial term of the perturbation series (including non-trivial integration over internal momenta) is calculated and turns out to be infinite. Here a good student protests: "Since everything is defined only perturbatively, then nothing has been defined at all!" The professor tries to defend himself: "But the theory has to be renormalized, and then the final results are in excellent agreement with experiments." It would hardly be impolite when the student replies: "Are you an experimentalist or a theorist? Renormalization is indeed necessary, but you must renormalize your style!"

This monograph was written to avoid such a disaster. This is achieved in the following way: We start with the classical Dirac theory of electrons and positrons in Chapter 1. In Chapter 2 the electron-positron field is quantized in external time-dependent electromagnetic (C-number) fields. In this case we find that the time evolution of the electron-positron field cannot be implemented, in general, in a fixed Fock Hilbert space. This forces us to abandon the usual lagrangian or hamiltonian approach. We retreat to scattering theory and first construct the (second quantized) S-matrix for the external field problem. The S-matrix is uniquely determined up to a phase. The second step is the determination of this phase by means of causality. The causal phase is related to vacuum polarization. Since this is one place where divergences appear in the usual formalism of QED, we are led to the conclusion that the correct incorporation of causality is the way to solve the ultraviolet divergence problems. That this is indeed true for full QED, too, follows from the work of Epstein and Glaser (*Annales de l'Institut Poincaré 29, p. 211, 1973*). We follow their method in Chapter 3 in the construction of the S-matrix of full QED by causal perturbation theory. The important point here is that this directly leads to the *renormalized* perturbation series. In fact, no divergent Feynman integral will appear in this book, explaining why the title "Finite QED" was chosen. In contrast to other approaches, ultraviolet finiteness is obtained here not as a result of a clever recipe but as a consequence of causality. Besides this conceptional advantage of the theory there is also a practical one because the actual computations of radiative corrections are simplified: only the minimum number of integrations need to be carried out, the non-trivial ones are dispersion integrals. The input of the dispersion relations is not extracted from an ill-defined Feynman integral, but is unambiguously given in terms of the lower orders of the perturbation series. Furthermore, the method is not restricted to one-loop calculations.

The work of Epstein and Glaser has sometimes been misunderstood as an elegant formulation of renormalization. Its importance in our opinion lies in the fact that it does away with renormalization. After the successes of renormalization group methods this point of view seems to stand orthogonal to the main stream of current research. However, we do not question the renormalization group as a technical tool. We only try to shake the dogma that renormalization is essential for understanding the foundations of field theory. What is essential is the correct manipulation of distributions. In fact, we find in our causal approach that *the ultraviolet problem is a consequence of incorrect splitting of distributions. The correct distribution splitting immediately gives the right finite ("renormalized") results.* Mathematicians may laugh and say that physicists simply made an error. The matter is not so simple. In the conventional lagrangian approach one must willy-nilly start from ill-defined quantities, then, making a lot of formal manipulations, but very late "after renormalization", everything becomes well defined. Where is "the error" then? There is none (or, at least, not many). The "error" can only be located in an approach where, from the very beginning, everything is well defined.

After the ultraviolet divergences have disappeared, the infrared problem becomes transparent: infrared divergences show up in the n-point functions $T_n\,(n \geq 3)$, if some particle momentum p approaches the mass shell ($p^2 = m^2$). However, these singularities are integrable such that T_n *as a distribution* is well defined. The weak singularities only remind us that T_n is a distribution and not a function. There is no need to modify the definition of the S-matrix or to introduce a finite photon mass. The singularities cancel out if the right physically measurable quantities are computed. One should not conclude from all these statements that QED is *completely* understood. This will not be the case until we have a non-perturbative construction of the S-matrix. But that is another story. The methods described in this monograph obviously apply to other relativistic quantum field theories, in particular to the electro-weak theory. But that is yet another story.

Zürich, 1988 *G. Scharf*

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