Die Grundlehren der mathematischen Wissenschaften

in Einzeldarstellungen
mit besonderer Berücksichtigung
der Anwendungsgebiete

Band 123

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Functional Analysis

Third Edition

Springer-Verlag Berlin Heidelberg GmbH

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Geschäftsführende Herausgeber:

Prof. Dr. B. Eckmann
Eidgenössische Technische Hochschule Zürich

Prof. Dr. B. L. van der Waerden
Mathematisches Institut der Universität Zürich

AMS Subject Classification (1970): 46-XX

ISBN 978-3-662-00783-9 ISBN 978-3-662-00781-5 (eBook)
DOI 10.1007/978-3-662-00781-5

Ursprünglich erschienin bei Springer-Verlag Berlin · Heidelberg 1971

Softcover reprint of the hardcover 3rd edition 1971

Preface to the First Edition

The present book is based on lectures given by the author at the University of Tokyo during the past ten years. It is intended as a textbook to be studied by students on their own or to be used in a course on Functional Analysis, i.e., the general theory of linear operators in function spaces together with salient features of its application to diverse fields of modern and classical analysis.

Necessary prerequisites for the reading of this book are summarized, with or without proof, in Chapter 0 under titles: Set Theory, Topological Spaces, Measure Spaces and Linear Spaces. Then, starting with the chapter on Semi-norms, a general theory of Banach and Hilbert spaces is presented in connection with the theory of generalized functions of S. L. SOBOLEV and L. SCHWARTZ. While the book is primarily addressed to graduate students, it is hoped it might prove useful to research mathematicians, both pure and applied. The reader may pass, e.g., from Chapter IX (Analytical Theory of Semi-groups) directly to Chapter XIII (Ergodic Theory and Diffusion Theory) and to Chapter XIV (Integration of the Equation of Evolution). Such materials as "Weak Topologies and Duality in Locally Convex Spaces" and "Nuclear Spaces" are presented in the form of the appendices to Chapter V and Chapter X, respectively. These might be skipped for the first reading by those who are interested rather in the application of linear operators.

In the preparation of the present book, the author has received valuable advice and criticism from many friends. Especially, Mrs. K. HILLE has kindly read through the manuscript as well as the galley and page proofs. Without her painstaking help, this book could not have been printed in the present style in the language which was not spoken to the author in the cradle. The author owes very much to his old friends, Professor E. HILLE and Professor S. KAKUTANI of Yale University and Professor R. S. PHILLIPS of Stanford University for the chance to stay in their universities in 1962, which enabled him to polish the greater part of the manuscript of this book, availing himself of their valuable advice. Professor S. ITO and Dr. H. KOMATSU of the University of Tokyo kindly assisted the author in reading various parts of the galley proof, correcting errors and improving the presentation. To all of them, the author expresses his warmest gratitude.

Thanks are also due to Professor F. K. SCHMIDT of Heidelberg University and to Professor T. KATO of the University of California at Berkeley who constantly encouraged the author to write up the present book. Finally, the author wishes to express his appreciation to Springer-Verlag for their most efficient handling of the publication of this book.

Tokyo, September 1964

KÔSAKU YOSIDA

Preface to the Second Edition

In the preparation of this edition, the author is indebted to Mr. FLORET of Heidelberg who kindly did the task of enlarging the Index to make the book more useful. The errors in the second printing are corrected thanks to the remarks of many friends. In order to make the book more up-to-date, Section 4 of Chapter XIV has been rewritten entirely for this new edition.

Tokyo, September 1967

KÔSAKU YOSIDA

Preface to the Third Edition

A new Section (9. Abstract Potential Operators and Semi-groups) pertaining to G. HUNT's theory of potentials is inserted in Chapter XIII of this edition. The errors in the second edition are corrected thanks to kind remarks of many friends, especially of Mr. KLAUS-DIETER BIERSTEDT.

Kyoto, April 1971

KÔSAKU YOSIDA

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