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Felix Papier

# Optimization of Rental Systems

Queuing Loss Theory for the Optimization of Cargo Vehicle Rental Systems

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Wiesbaden, Germany

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# List of Abbreviations

ADI	Advance demand information
ADIP	ADI policy
AI	Arbitrary arrivals and independent service times
al.	Alii
Arb.	Arbitrary
ASA	Average stationary approximation
B2B	Business to business
BMP	Benchmark policy
C.	Coils
cdf	Cumulative distribution function
cf.	Confer
Ch.	Chapter
CTMC	Continuous-Time Markov chain
DFMP	Dynamic fleet management problem
DTMC	Discrete-Time Markov chain
e.g.	Exempli gratia
EBB	Entire batch blocking
eDTMC	Embedded discrete-time Markov chain

ELM	Erlang loss model
Eq.	Equation
Ex.	Exponential
Exp.	Expected
GPS	Global positioning system
i.e.	Id est
i.i.d.	Identically and independently distributed
Ineq.	Inequality
IP	Independence properties
mio.	Million
n/a	not applicable
Par./Params.	Parameters
PASTA	Poisson arrivals see time averages
PBB	Partial batch blocking
PC	Personal computer
pdf	Probability distribution function
Poi.	Poisson
Reg.	Regulars
Rev.	Revenues
ROS	Return on sales
Sim.	Simulation
SL	Service level
SLAP	Successive linear approximation procedure
U.S.	United States
UB	Upper bound
vs.	Versus
w/o	Without
yr	Year

# List of Symbols

$\mathbf{0}$	Null matrix
$\bar{\mathbf{0}}$	Null vector
$1()$	Indicator function
$\alpha()$	Auxiliary function
$a$	Admission decision
$A$	Arrival time
$A$	Arrival process, auxiliary matrix (Ch. 5)
$\beta$	Blocking probability
$b$	Discrete auxiliary variable
$B$	Batch size, blocking probability (Ch. 3)
$B$	Submatrix of $Q$
$c$	Fleet size
$C$	Auxiliary matrix
$CL$	Class indicator
$\delta$	Row sum of a matrix
$\Delta$	Difference
$D$	Diagonal matrix with eigenvalues on the diagonal
$D$	Departure event



$E()$	Expected value
$\varepsilon$	Continuous auxiliary variable
$f()$	Probability distribution function
$f^{a*}()$	$a$ -Fold convolution of $f()$
$F()$	Cumulative distribution function
$\bar{F}()$	Complementary cumulative distribution function
$\Gamma$	Admission policy
$g()$	Probability distribution function
$G()$	Cumulative distribution function
$h$	Unit holding cost
$i$	Discrete auxiliary variable
$I$	Difference in system states
$I_n$	Identity matrix of size $n$
$j$	Discrete auxiliary variable
$J(t)$	Service completions before time $t$
$k$	Discrete auxiliary variable
$K(t)$	Service completions before time $t$
$l_t$	Leasing quantity in period $t$
$l$	Vector of leasing quantities
$L$	Exp. lost sales (Ch. 2), loss in profit (Chs. 4 and 5)
$\mu$	Reciprocal of mean rental time
$N$	Number of busy cars
$\phi()$	PDF of standard normal distribution
$\Phi()$	CDF of standard normal distribution
$\pi$	State probability
$\pi$	State distribution

$\Pi$	Exp. profit
$\psi()$	Auxiliary function
$p$	Unit penalty cost
$p_i$	Probability of event $i$
$P()$	Probability
$\tilde{P}()$	Probability generating function
$P(t)$	Transition matrix after $t$ time units
$q$	Mean order size
$Q$	Intensity matrix
$\tilde{Q}$	Submatrix of $Q$
$\hat{Q}$	Submatrix of $Q$
$\lambda$	Arrival rate of Poisson process
$r$	Revenue margin
$R$	Set of advance demand information
$\sigma$	Standard deviation
$S$	System
$S$	Rental or service time
$SL$	Service level
$\tau$	Demand leadtime
$t$	Time epoch
$\hat{t}$	Time epoch
$T$	Number of periods (Ch. 2), threshold level (Chs. 4 and 5)
$u$	Auxiliary vector
$\tilde{\mathbf{u}}$	Auxiliary vector
$U$	Uniformly distributed random number on $[0, 1]$
$U$	Matrix of eigenvectors

$v$	Unit leasing cost
$\mathbf{v}$	Auxiliary vector
$\tilde{\mathbf{v}}$	Auxiliary vector
$V$	Served requests (Ch. 2), exp. service rate (Chs. 4 and 5)
$w$	Weight variable (Ch. 4), eigenvalue (Ch. 5)
$x$	Continuous auxiliary variable
$x$	Auxiliary vector
$X$	Departure event (Ch. 5)
$X_n$	Time epoch of $n$ th service completion (Ch. 3)
$Y_n$	Time epoch of $n$ th service completion
$z$	Continuous auxiliary variable