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Manuela Spangler

# Modelling German Covered Bonds

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# Abstract

With a long track record going back to the eighteenth century and no single case of default until today, the German Pfandbrief has an undisputed benchmark status in the covered bond market. It survived the recent financial and sovereign crises comparably unharmed and has proved to be a reliable and stable funding instrument also in times of market distress. Nevertheless, as the past with several Pfandbrief bank bailouts has shown, Pfandbriefe cannot be considered to be completely risk-free, despite their high level of protection. To adequately model the risks arising from a Pfandbrief investment, it is not sufficient to consider only the creditworthiness of the issuer. Product-specific features and the quality of the cover pool also need to be taken into account.

In this work we develop a multi-period simulation-based Pfandbrief model which accounts for the product's most important characteristics and adequately reflects its main risks. The model distinguishes between bank and cover pool default and considers two different default triggering events: overindebtedness and illiquidity. Both default events are influenced by the market environment, which is represented through the stochastic dynamics of the short rate and the creditworthiness of the bank's risky assets, and the resulting liquidation payments take into account the Pfandbrief-specific priority of payments. The asset liability management in our model is dynamic and considers funding and reinvestment strategies as well as the maintenance of overcollateralization according to the legal requirements.

The model's primary outputs are Pfandbrief default statistics. Simulation results obtained from an exemplary Mortgage Pfandbrief calibration with typical asset liability mismatches capture the main expected behaviour patterns of Pfandbriefe. Due to its modular setup, our model provides a flexible framework for structural analyses and can be easily extended for tailor-made investigations. Potential areas of applications include but are not limited to the comparison of different Pfandbrief risk profiles and studies in the context of current policy debates such as the introduction of extendible Pfandbrief maturities.