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Foundations of Synergetics II

Complex Patterns

With 98 Figures

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Preface

This textbook is based on a lecture course in synergetics given at the University of Moscow. In this second of two volumes, we discuss the emergence and properties of complex chaotic patterns in distributed active systems. Such patterns can be produced autonomously by a system, or can result from selective amplification of fluctuations caused by external weak noise.

Although the material in this book is often described by refined mathematical theories, we have tried to avoid a formal mathematical style. Instead of rigorous proofs, the reader will usually be offered only “demonstrations” (the term used by Prof. V. I. Arnold) to encourage intuitive understanding of a problem and to explain why a particular statement seems plausible. We also refrained from detailing concrete applications in physics or in other scientific fields, so that the book can be used by students of different disciplines.

While preparing the lecture course and producing this book, we had intensive discussions with and asked the advice of Prof. V. I. Arnold, Prof. S. Grossmann, Prof. H. Haken, Prof. Yu. L. Klimontovich, Prof. R. L. Stratonovich and Prof. Ya. B. Zeldovich. We wish to express our gratitude to all of them. We are especially grateful to Prof. Ya. G. Sinai who read the chapters on chaotic dynamics and made important comments. We also wish to thank our students who posed naive questions and made sharp criticisms, and thus contributed very much to the improvement of this text. Most of the graphic illustrations in this volume were produced by one of the authors (A. Yu. L.).

Moscow and Stuttgart, March 1991

A. S. Mikhailov, A. Yu. Loskutov

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