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Structural Complexity I

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Preface

Since the achievement of a formal definition of the concept of “algorithm”, the Mathematical Theory of Computation has developed into a broad and rich discipline. The notion of “complexity of an algorithm” yields an important area of research, known as Complexity Theory, that can be approached from several points of view. Some of these are briefly discussed in the Introduction and, in particular, our view of the “Structural” approach is outlined there.

We feel the subject is mature enough to permit collecting and interrelating many of the results in book form. Let us point out that a substantial part of the knowledge in Structural Complexity Theory can be found only in specialized journals, symposia proceedings, and monographs like doctoral dissertations or similar texts, mostly unpublished. We believe that a task to be done soon is a systematization of the interconnections between all the research lines; this is a serious and long task. We hope that the two volumes of this book can serve as a starting point for this systematization process.

This book assumes as a prerequisite some knowledge of the basic models of computation, as taught in an undergraduate course on Automata Theory, Formal Language Theory, or Theory of Computation. Certainly, some mathematical maturity is required, and previous exposure to programming languages and programming techniques is desirable. Most of the material of Volume I can be successfully presented in a senior undergraduate course; Volumes I and II should be suitable for a first graduate course. Some sections lead to a point in which very little additional work suffices to be able to start research projects. In order to ease this step, an effort has been made to point out the main references for each of the results presented in the text.

Thus, each chapter ends with a section entitled “Bibliographical Remarks”, in which the relevant references for the chapter are briefly commented upon. These sections might also be of interest to those wanting an overview of the evolution of the field. Additionally, each chapter (excluding the first two, which are intended to provide some necessary background) includes a section of exercises. The reader is encouraged to spend some time on them. Some results presented as exercises are used later in the text; however, this is the exception, not the rule. Many exercises are devoted to presenting a definition and some properties of an interesting concept. Usually, a reference is provided for the most interesting and for the most useful exercises. Some exercises are marked with a • to indicate

that, to the best knowledge of the authors, the solution has a certain degree of difficulty.

This book originated from a somewhat incomplete set of lecture notes for a series of lectures given at the Institute of Computer Science and Cybernetics of Hanoi, Vietnam, during the summer of 1985. We would like to thank the head of the Institute at the time, Dr. Phan Dinh Dieu, for the invitation, as well as for the comments and remarks made during the lectures.

We have benefited from conversations with many friends and colleagues. In particular we would like to express our gratitude to Ron Book, Rafael Casas, Uwe Schöning, and many other colleagues for their suggestions and help, and to Peter van Emde Boas, Jacobo Torán, Carme Torras, the students of our Department, and an anonymous referee for pointing out many corrections which have improved the quality of the manuscript. We also would like to thank Rosa Martín and the other staff of the Laboratori de Càlcul de la Facultat d'Informàtica de Barcelona, for their logistic help in installing T_EX on our computer system.

Barcelona, October 1987

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