WAVE PROPAGATION IN DISSIPATIVE MATERIALS

A REPRINT OF FIVE MEMOIRS

BY

B. D. COLEMAN, M. E. GURTIN, I. HERRERA R., AND C. TRUESDELL

SPRINGER-VERLAG · NEW YORK INC. 1965

ISBN 978-3-642-88693-5 ISBN 978-3-642-88691-1 (eBook)
DOI 10.1007/978-3-642-88691-1

Softcover reprint of the hardcover 1st edition 1965

Library of Congress Catalog Card Number 65-25011

Title No. 1306

Preface

Common experience reveals two basic aspects of wave propagation. First, while preserving their identity and travelling at definite speeds, sounds finally die out. Second, weak sounds may combine to form strong noises. Theories of acoustic propagation have succeeded in representing these aspects of experience separately, but never combined as in nature. The classical theories of sound in perfect fluids and elastic solids easily yield common speeds of propagation for plane infinitesimal disturbances, but no damping. Moreover, within EULER's theory of the perfect fluid, or its generalization, the GREEN-KIRCHHOFF-KELVIN theory of finite elasticity, weak waves may grow stronger and become shock waves, which propagate according to more complicated but equally definite principles. Effects of internal damping are easily added for theories of infinitesimal deformation, but for finite motions a dead end was reached about sixty years ago.

Indeed, in 1901 DUHEM proved that according to the NAVIER-STOKES theory of fluids acceleration waves and waves of higher order cannot exist, and for shock waves he claimed a similar result, which has since been shown to be valid subject to certain qualifications. So as to save the phenomena of sound and noise, as was necessary if the NAVIER-STOKES theory was to deserve the place proposed for it as a refinement upon EULER's theory, DUHEM introduced the concept of "quasi-wave", a region of rapid but continuous transition. In 1906 PRANDTL offered the same argument in a more special and less precise way, and from his remarks grew, eventually, not only the theory of the plane "shock layer" with its notorious mathematical difficulties, but also the widespread opinion that any kind of "dissipative mechanism" smoothes out discontinuities.

In 1930 LAMPARIELLO observed that this opinion is false. Indeed, the linear hyperbolic partial differential equation

$$T \frac{\partial^2 u}{\partial x^2} + F \frac{\partial u}{\partial t} = \sigma \frac{\partial^2 u}{\partial t^2},$$

which represents the small transverse motion of a perfectly flexible string with a frictional resistance proportional to the velocity, clearly *admits discontinuous solutions of all orders*. Moreover, their speed of propagation is $U = \sqrt{T/\sigma}$, unaffected by the magnitude of the coefficient of friction F. The idea is easily generalized, and in the vast Italian literature on accumulative theories ("fisica ereditaria"), little or no attention has been paid to waves, since in the equations considered there certain accumulative terms ("termini ereditari") involving only derivatives of order lower than that of the differential system are added to account for dissipation. Of course, with shock waves the argument no longer applies, but the theories are linearized from the start, with only infinitesimal deformations in view, so the question need not arise.

The non-Italian literature, meanwhile, continued to regard surfaces of discontinuity incompatible with "dissipative mechanisms" until just the last few years, when solutions involving shock waves according to BOLTZMANN'S theory of infinitesimal visco-elasticity began to be noticed and discussed.

The scene was now set for the four remarkable memoirs reprinted in this volume. These provide the first theory of waves that accounts for *all common acoustic experience.*

Essential to the achievement here recorded was the general development of continuum mechanics in the past twenty years, and in particular the following three mathematical structures. (1) The theory of simple materials as formulated by NOLL in 1957—8. (2) The theory of fading memory as defined and applied by COLEMAN & NOLL in 1959—60. (3) The thermodynamics of simple materials as created by COLEMAN in 1964. Very helpful also was the development of technique in the theory of singular surfaces in non-dissipative media by T. Y. THOMAS in the late forties and early fifties, followed by ERICKSEN'S definitive memoir of 1953 on waves in incompressible isotropic elastic materials.

While this whole complex of ideas and the associated mathematics are used here, the key to resolution of the apparent paradox of wave propagation in dissipative materials may be found in the fact that although the NAVIER-STOKES theory emerges according to COLEMAN & NOLL'S scheme as an *asymptotic approximation* in the limit of slow motion for any simple fluid with fading memory, it is itself an exceptional material in that it does *not* exhibit fading memory in COLEMAN & NOLL'S sense. COLEMAN, GURTIN & HERRERA have proved that the property of fading memory *removes the inconsistency between sharp discontinuities and dissipation*, so that, in accord with experience, *both damping and propagation* become possible in theories of materials. The inability of the NAVIER-STOKES fluid to support wave motions merely illustrates its exceptional character, untypical of real fluid behavior. Looking backward now, we can say that the theory of linearly viscous fluids, although properly invariant, was misleading as a basis for the kind of ritual which is called "intuition" by those who consider themselves innately endowed with knowledge of physics. BOLTZMANN'S theory of infinitesimal visco-elastic materials is closer to nature in regard to wave propagation, yet in turn untypical because of the linearization, which renders all kinds of waves equivalent and removes the possibility of reinforcement. The theory of COLEMAN, GURTIN, & HERRERA unifies the HUGONIOT-HADAMARD theory of weak and strong waves in finitely deformed elastic fluids and solids with the recent studies of waves in infinitesimal visco-elasticity and extrapolates between them as limit cases; it takes account both of finite deformations and of long-range memory.

There is not space here to summarize the papers reprinted in this volume. I confine myself to pointing out two remarkable results. First, for a given material there exists a critical jump in the density rate [Remark 5.2 in Part II (p. 255)]: If the instantaneous modulus of compression is positive, a plane compressive acceleration wave carrying a lesser jump is damped out steadily as it progresses into a region at rest, but the amplitude of a wave carrying a greater jump becomes infinite within a finite time. In Part III this result is shown to remain valid when thermodynamic influences are taken into account. Thus appears, for the first time in a mathematical theory, an explanation for the damping of sufficiently weak sounds and the explosion of sufficiently strong ones. Second, in Part IV thermodynamic limitations are used to show that the acoustic tensor in a non-conductor or a definite conductor of heat is a symmetric tensor, a result which suggests an experimental test of the theory here developed. I remark also that the general theory presented here puts singular surfaces back in place as *the best model* for propagation of nearly all kinds of sharp disturbances, except, of course, in cases where the finite thickness of a region of transition is really of prime interest.

When I proposed the immediate reprinting of these four memoirs on the general theory, Messrs. COLEMAN and GURTIN asked me to include my paper of 1961 on waves in elastic materials. While its subject is strictly excluded by the title of the volume, it may be useful as a reference, since in many cases the theorems on waves in dissipative materials are phrased by reduction to the theory of waves in a particular purely elastic material, determined by the constitutive functional and the deformation-temperature history together. For the reprinting, a few misprints and slips have been corrected, and a few remarks, set off in braces, have been added. May the inclusion of my paper, with its dedication and references, remind the reader of the great tradition of HUGONIOT, HADAMARD, and DUHEM, here vindicated and refurbished.

This preface must close by an expression of gratitude to Springer-Verlag, not only for its peerless typography and accuracy, but also for the elegant courtesy, vanishing relic of old times, which it extends to its authors and editors.

C. TRUESDELL

Contents

Reprinted from Volume 8 (1961)
Archive for Rational Mechanics and Analysis

Reprinted from Volume 19 (1965)
Archive for Rational Mechanics and Analysis