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in Einzeldarstellungen
mit besonderer Berücksichtigung
der Anwendungsgebiete

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Preface

The development of the internationally standardized language ALGOL has made it possible to prepare procedures which can be used without modification whenever a computer with an ALGOL translator is available. Volume Ia in this series gave details of the restricted version of ALGOL which is to be employed throughout the Handbook, and volume Ib described its implementation on a computer. Each of the subsequent volumes will be devoted to a presentation of the basic algorithms in some specific areas of numerical analysis.

This is the first such volume and it was felt that the topic Linear Algebra was a natural choice, since the relevant algorithms are perhaps the most widely used in numerical analysis and have the advantage of forming a well defined class. The algorithms described here fall into two main categories, associated with the solution of linear systems and the algebraic eigenvalue problem respectively and each set is preceded by an introductory chapter giving a comparative assessment.

In spite of the self-contained nature of the linear algebra field, experience has shown that even here the preparation of a fully tested set of algorithms is a far greater task than had been anticipated. Almost all the algorithms presented here have received pre-publication in *Numerische Mathematik* and the need to check very carefully whenever an algorithm is modified, even in a minor detail, has meant that the preparation of a paper in the Handbook series has usually taken longer than would be required for a normal research paper. Failure to check may result in the introduction of an error into an algorithm which previously had enjoyed a long period of successful use.

It soon became obvious that it would be impractical, even if desirable, to aim at completeness in this volume. In general we have aimed to include only algorithms which at least in some limited area provide something approaching an optimum solution, this may be from the point of view of generality, elegance, speed or economy of storage. The omission of an algorithm should not be taken as indicating that it has been found wanting; in some cases it merely means that we were not fully satisfied with current implementations.

From its very nature a volume of this kind is essentially a joint effort. In many instances the basic mathematical concept of an algorithm has had a long period of evolution and many people have played a part in the progressive refinement of the original concept. Thanks are due to all who have thereby contributed indirectly to this volume but I would like to pay a special tribute to those who have submitted the ALGOL procedures published here and have assisted in testing them.

Professor F. L. Bauer and the late Professor H. Rutishauser, who are authors of the Handbook series, have been invaluable sources of information and have supervised much of the work in this volume. Three colleagues at the National

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Physical Laboratory, Miss. H. Bowdler, Mr. R. S. Martin and Mrs. G. Peters have between them been responsible for the preparation of some half of the published algorithms, have made substantial contributions to them and have played a major part in their testing at all stages in their development. Special thanks are due to Mrs. E. Mann of the Technische Universität München, who tested a number of the earlier variants and virtually all of the final versions of the algorithms and gave valuable assistance in the proof reading.

The publication of a volume of this kind has special problems since a good deal of the typeset material has had to be retained in almost finished form over a long period. We are particularly grateful to the publishers for their patient courteous assistance in what must have been an unusually exacting task.

We are very much aware that the production of reliable algorithms is a continuing process and we shall appreciate it if users will inform us of failures or shortcomings in the performance of our procedures.

Teddington and München
October 1970

J. H. Wilkinson
C. H. Reinsch

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General guide to the use of the volume

Most of the contributions have been subdivided into the following seven sections.

1. Theoretical Background
briefly collecting the relevant formulae.
2. Applicability
which sketches the scope of the algorithms.
3. Formal Parameter List
with a specification of all input, output and exit parameters.
4. ALGOL Programs
presenting the algorithms in the ALGOL 60 reference language.
5. Organisational and Notational Details
explaining and commenting on special features, and showing why particular realizations of the formulae have been preferred to others.
6. Discussion of Numerical Properties
usually referring to a published error analysis of the underlying algorithms.
7. Test Results and Examples of the Use of the Procedures
giving the solutions obtained on a digital computer to a number of sample problems in order to facilitate the correct use of the procedures.

In some earlier contributions section 7 above was subdivided into two sections:

7. Examples of the Use of the Procedures.
8. Test Results.

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