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Symmetries, Topology and Resonances in Hamiltonian Mechanics

Springer

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Mephistopheles
Ich wünschte nicht, Euch irrezuführen.
Was die Wissenschaft betrifft,
Es ist so schwer, den falschen Weg zu meiden,
Es liegt in ihr so viel verborgenes Gift,
Und von der Arzenei ists kaum zu unterscheiden.
Am besten ists auch hier, wenn Ihr nur Einen hört
Und auf des Meisters Worte schwört.
Im ganzen: haltet Euch an Worte !
Dann geht Ihr durch die sichre Pforte
Zum Tempel der Gewißheit ein.

Schüler
Doch ein Begriff muß bei dem Worte sein.

Faust I. Goethe. Goethe Werke B. 3.
Insel Verlag, Frankfurt am Main 1977

Preface

The problem of exact integration of equations of dynamics has been one of the most popular fields of research since Newton's famous "Philosophiae naturalis principia mathematica". The principal idea in the investigation is the general notion of symmetry. When solving the problem of a mass point motion in a central field, Newton already used the concept of symmetry: by factorizing orbits of the rotation group he reduced this problem to a one-dimensional motion in a potential field. Later, Lagrange and Jacobi noticed that the classical integrals of the n-body problem are related to invariance of the equations of motion with respect to Galileo's transformation group. This fundamental remark was generalized by Emmy Noether: to each transformation group conserving the Hamiltonian action there corresponds an integral of the motion equations. The converse is also correct: a phase flow of a Hamiltonian system with a known additional integral as a Hamiltonian takes solutions of the original equations of motion into solutions of the same equations. This gives the idea of the proof of the well-known Liouville theorem on the complete integrability of Hamilton's equations: the phase flows of involutive integrals commute pairwise and generate an Abelian symmetry group of maximal possible dimension on their joint level manifolds.

At first, the question of exact integration was treated only in an analytical way: to find explicit expressions for first integrals and solutions. However, after Poincaré's work it became clear that the integrability phenomenon is closely connected with the global behavior of phase trajectories. When studying a dynamical system "in the whole" it is essential to know its topology. Quite recently it was discovered that the complicated topological structure of the configuration space is not consistent with integrability of the motion equations of the corresponding dynamical system. On the other hand, as shown by Poincaré, obstructions to integrability of Hamiltonian systems are resonance phenomena connected with the destruction of invariant resonant tori under perturbations. An analytical aspect of this fact is the well-known problem of small divisors in celestial mechanics. The other known obstacles to integrability – splitting of asymptotic surfaces and branching of solutions in the complex time plane – are also closely connected with resonances.

This book makes the first attempt to systematize the results on integrability of Hamiltonian systems obtained during the past 10–15 years as well as to give a modern interpretation of classical findings in this field.

The contents of the book are as follows. In the Introduction we give a historical survey of the integrability problem in dynamics. The basic notions of Hamiltonian mechanics are explained in Chapter I. Chapter II is devoted to the methods of exact integration of Hamiltonian systems: here we discuss various concepts of integrability of such systems. In Chapter III we indicate rough obstructions to integrability expressed in terms of topological invariants of the configuration space. In Chapters IV-VIII we discuss resonance phenomena in connection with the integrability problem. The methods given there enable one to prove rigorously the nonintegrability of many urgent problems of dynamics. Special attention is given to the phenomenon of stochastization of Hamiltonian systems under a small perturbation of the Hamiltonian function.

Our explanations use various mathematical techniques. However, those notions that go beyond the standard university course can be found in the book itself. Therefore, reader's persistence and patience is all that is needed. The book is intended primarily for young mathematicians and physicists who have the possibility of advancing this fascinating field, which has many important problems still unsolved.

Moscow, Summer 1995 Valerij V. Kozlov

Table of Contents