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Introduction to the Theory of (Non-Symmetric)

Dirichlet Forms

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Z.-M. M. dedicates this book to his wife.

M. R. dedicates this book to his father
on the occasion of his 65th birthday.

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