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Stephen Maybank

Theory of Reconstruction from Image Motion

With 29 Figures

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Preface

The image taken by a moving camera changes with time. These image motions contain information about the motion of the camera and about the shapes of the objects in the field of view. There are two main types of image motion, finite displacements and image velocities. Finite displacements are described by the point correspondences between two images of the same scene taken from different positions. Image velocities are the velocities of the points in the image as they move over the projection surface. Reconstruction is the task of obtaining from the image-motions information about the camera motion or about the shapes of objects in the field of view. In this book the theory underlying reconstruction is described.

Reconstruction from image motion is the subject matter of two different scientific disciplines, photogrammetry and computer vision. In photogrammetry the accuracy of reconstruction is emphasised; in computer vision the emphasis is on methods for obtaining information from images in real time in order to guide a mechanical device such as a robot arm or an automatic vehicle. This book arises from recent work carried out in computer vision. Computer vision is a young field but it is developing rapidly. The earliest papers on reconstruction in the computer vision literature date back only to the mid 1970s. As computer vision develops, the mathematical techniques applied to the analysis of reconstruction become more appropriate and more powerful. The advances in the theory are matched by the advances in the speed and capacity of the electronic equipment available for computer vision. It is now possible to implement in real time methods which were of only theoretical interest ten years ago.

The book is divided into six chapters. Chapter 1 is an introduction. Chapters 2 and 3 describe reconstruction from image correspondences, with particular emphasis on the ambiguous case, in which the same set of image correspondences yields two or more different reconstructions. Chapter 4 describes reconstruction from image velocities, again with an emphasis on the ambiguous case. Chapter 5 describes cases in which the data for reconstruction are just sufficient to ensure that only a finite number of reconstructions are possible. In Chap. 6 four algorithms for reconstruction from image motions are described. Two of the algorithms require image correspondences, and the remaining two require image velocities.

The chapters are divided into sections and the sections are divided into subsections. For example Sect. 2.1.1 is Subsection 1 of Section 1 of Chapter 2. Most of the results are stated either as theorems or as propositions. A proposition is usually less important or easier to prove than a theorem. The theorems, propositions and definitions are labelled in sequence, chapter by chapter. The numbering of the sections and subsections within a chapter is independent of the numbering of the theorems. The end of a proof of a theorem or a proposition is marked by a box, □, at the right-hand margin of the final line. The references are placed at the end of each chapter.

To return to the contents of the chapters in more detail: Chapter 1 describes the background to reconstruction. Certain useful mathematical concepts and notation are introduced. Two different but mathematically equivalent frameworks for reconstruction from point correspondences are described in Chap. 2, namely the Euclidean framework and the projective geometric framework. The Euclidean framework emphasises the description of the image in terms of coordinates, whilst the projective geometric framework emphasises the description of the image in terms of the lines in space passing through the optical centre of the camera. The ambiguous case of reconstruction is discussed within both frameworks. The properties of certain 3×3 matrices known as essential matrices are described. These matrices arise naturally in the Euclidean framework. They summarise the information about the camera displacement contained in a set of image correspondences. Reconstruction up to a collineation is discussed briefly at the end of Chap. 2. In this approach the full camera calibration is not required, but the reconstruction has a higher degree of ambiguity.

The geometry of the ambiguous case of reconstruction from image correspondences is developed in Chap. 3. Ambiguity is only possible if the points giving rise to the image correspondences lie on certain surfaces of degree two known as critical surfaces. There are at least two critical surfaces involved in each case of ambiguity, one for each possible reconstruction. The intersection of two critical surfaces contains a curve known as a horopter curve. Many properties of critical surfaces are directly related to the properties of horopter curves. The critical surfaces for the ambiguous case of reconstruction up to a collineation are discussed.

Chapter 4 describes the reconstruction of camera velocity from image velocities. It is shown that the ambiguous case is similar in many respects to the ambiguous case of reconstruction from image correspondences. The constraints on the camera velocity arising from four image velocity vectors are examined in detail. The translational velocity is subject to a quartic polynomial constraint. If the image velocity field is irregular then the leading order terms of the quartic split into two linear factors and a quadratic factor.

Chapter 5 discusses three cases of reconstruction in which the data obtained from the images are just sufficient to ensure that only a finite number of reconstructions are possible. The three cases are reconstruction from point correspondences, reconstruction from image velocities and reconstruction up to a collineation. In these minimal cases the algebraic nature of reconstruction is particularly apparent. Reconstruction is formulated as the problem of finding the common zeros of a particular set of multivariate polynomials. The degree of a reconstruction problem is defined to be the number of reconstructions obtained in the minimal case. If the degree is high then reconstruction is likely to be intrinsically difficult, regardless of the particular algorithm employed. The degree of reconstruction from point correspondences and from image velocities is ten in both cases. The degree of reconstruction up to a collineation is three.

In Chapter 6, four different reconstruction algorithms are described. Two are for reconstruction from image correspondences, and two are for reconstruction from image velocities. In all four algorithms it is assumed that the camera calibration is known. The algorithms for reconstruction from image velocities illustrate the importance of irregularities in the image velocity field. If the moving surface has a large change in depth or orientation over a small part of the field of view then reconstruction can be carried out using algorithms that are simpler than those designed for the general case. The results obtained are also more stable in the presence of noise.

The thesis on which part of this book is based began with a quotation from Machado suggested by Mari Carmen, 'Caminante no hay camino. Se hace camino al andar'. (Traveller there are no paths, Paths are made by walking.) Like a path, science is constructed, but at the same time it is not made arbitrarily. It has to *go* somewhere.

Sophia Antipolis
January 1992

S.J. Maybank

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