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Wei-Bin Zhang

# Synergetic Economics

Time and Change in Nonlinear Economics

With 92 Figures

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*To my Parents*

*who have suffered much from my having  
been away from home for so long*

. . . if orthodox economics is at fault, the error is to be found not in the superstructure, which has been erected with great care for logical consistency, but in a lack of clearness and of generality in the premises.

*J. M. Keynes (1936)*

# Foreword

This is a book about the dynamics of economic and other social systems. It was written at the Swedish Institute for Futures Studies and is oriented towards the problem of understanding economic evolution and rapid structural change.

The analysis is very closely related to synergetics. This implies that Dr. Zhang has focused considerable attention on the fact that economic and other social variables can be subdivided into subsets regulated by *slow* and *fast* processes, respectively. Some of the slow variables turn out to be of collective importance, i.e. they tend to act as order parameters of economic and social systems.

More or less mathematically, such a distinction has also been made in earlier analyses of dynamic economics. Alfred Marshall made the distinction in his nineteenth century textbook and Paul Samuelson did likewise in his *Foundations of Economic Analysis* in the 1940s. But they did not see the possibility of explicit solutions to the important dynamic economic problems implicit in such an approach to economics. Dr. Zhang not only points in the same direction, he also shows how the synergetic approach works in long term dynamic analyses of important development problems. One of the most important conclusions is that, with a proper subdivision into fast and slow interactive subsystems, predictability can be achieved in a system which would otherwise be unpredictable, i.e. chaotic. Furthermore, the analysis shows that certain order-enforcing variables are suitable as strategic policy instruments available for policy making. Most of these variables are slowly changing and can thus be seen as order parameters at the level of the economic system. This characteristic automatically implies that they become part of strategic decision making, i.e., instruments of future-oriented policies.

Studies of the future are clearly of importance; however, they can easily become utopian daydreaming if not based upon a stringent methodological foundation. With this book Dr. Zhang has provided one of the pillars of such a foundation.

Åke E. Andersson  
Professor of Economics at Umeå University  
and Director of the Swedish Institute for Futures Studies

# Preface

With the passage of time, not only does economic life change, but there is even a shift in the dominant economic ideas. While the classical economists, such as Smith, Ricardo, Malthus, Marx, Mill, Walras, and Marshall, have had and will have their days in different cultures at different times, it is still too early to judge the historical significance of contributions to economics by modern economists. Time is the arbiter of truth. Only time can make us wise enough to recognize the superficiality of ideas which initially sounded important and far reaching.

Not only the general public, but even many economists are tending to lose confidence in the applicability of economics to reality, although the level of knowledge of economics has increased greatly in recent years: there seems to be no simple relation between knowledge of and confidence in science.

One may conceive of different reasons why economics fails to explain reality. On the one hand, the complexity of the real world has increased dramatically in recent decades. For instance, technology, institutions, values and goals of human life, and morals, which were relatively slow to change in the past, are now subject to change in the short term as well. This character of modern society makes it difficult, if not impossible, for pure economics to accurately explain the complexity of economic life. On the other hand, traditional economics also has intrinsic limitations. For instance, traditional economics has been mainly limited to static or stabilized economic systems. Nonlinear unstable phenomena such as regular and irregular oscillations, which are the main concern of this study, have been considered to be temporary or insignificant in traditional economic analysis.

This book studies problems related to time and change in nonlinear unstable economic systems. We will concentrate on particular aspects of dynamic economic systems, for example nonlinearity, instability, bifurcations and chaos. We propose a new theory – “synergetic economics” – based on Haken’s synergetics, for analyzing the characteristics of nonlinear dynamic economic systems. Fundamentally, synergetic economics emphasizes the interplay between linearity and nonlinearity, stability and instability, continuity and discontinuity, permanence and structural change, in contrast to pure linearity, stability, continuity and permanence in economic evolutionary processes. Synergetic economics treats nonlinearity and instability as sources of the variety and complexity of economic dynamics, rather than as a nuisance and as temporary phenomena as traditional economics did.

In a sense, this book aims to complete the task which was suggested by Paul A. Samuelson when he wrote his celebrated *Foundations of Economic Analysis*. He broadly classified the development of analytical economics into five steps. First, in



Walras we have the final culmination of the notion of determinacy of equilibrium and the static level. Pareto and others took a second step, which laid the basis of a theory of comparative statics. The third step, which is characterized by maximizing action within an economic unit, was mainly carried out by Johnson, Slutsky, Hicks and Allen. The fourth advance is due to the discovery of the correspondence principle. "A natural fifth step to take after we have investigated the response of a system to change in given parameters is to investigate its behavior as a result of the passage of time." Furthermore, Samuelson emphasized that "The usefulness of any theoretical structure lies in the light which it throws upon the way economic variables will change when there is a change in some datum or parameter. This commonplace holds as well in the realm of dynamics as in statics. It is a logical next step, therefore, to begin to create a theory of comparative dynamics. This will include the theory of comparative statics as a special case, and indeed all of the earlier five subjects, but it will cover a much richer terrain" (Samuelson 1946). The fifth step will be cultivated in this book.

This book is suitable for students and researchers in economics. It may also be useful to scholars interested in applications of nonlinear dynamic theory to economic problems.

Stockholm, July 1990

*W.B. Zhang*

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# Contents

<b>1. Introduction</b> .....	1
<b>2. Time and Change in Economics</b> .....	7
2.1 Economic Evolution – An Introduction .....	7
2.2 Equilibrium Theories in Economic Analysis .....	8
2.3 Dynamic Theories in Economics .....	9
2.4 Samuelson’s Correspondence Principle and Its Limitations ....	11
2.5 Instabilities in Economic Analysis .....	13
<b>3. Mathematical Aspects of Dynamic Systems</b> .....	15
3.1 Dynamics and Equilibrium .....	15
3.2 Classifications of Two-Dimensional Differential Equations ....	20
3.3 The Principle of Linearized Stability .....	22
3.4 Lyapunov’s Direct Method .....	25
3.5 Structural Stability .....	27
3.6 Conservative Systems .....	30
3.7 Bifurcation Theory .....	34
3.8 Singularity Theory .....	40
3.9 Catastrophe Theory .....	43
Appendix: Remarks on Bifurcation Theory .....	46
<b>4. Multiple Equilibria and Structural Changes in Economic Systems</b> .....	48
4.1 Catastrophe Theory and Comparative Statics Analysis .....	48
4.2 Modeling Regional Dynamics .....	53
4.3 Some Examples of Structural Changes .....	55
4.3.1 Business Cycles in the Kaldor Model .....	56
4.3.2 Resource Management .....	57
4.3.3 Dynamic Transportation Modal Choice and Bifurcation .....	58
4.3.4 Multiple Equilibria in Wilson’s Retail Model .....	59
4.4 A Bifurcation Analysis for an Economic Growth Model .....	61
4.5 Singularity Theory in Economic Analysis .....	66
4.6 Remarks .....	67
<b>5. Economic Cycles</b> .....	68
5.1 Theories of Economic Cycles .....	68
5.2 Some Mathematical Results Related to Limit Cycles .....	72

5.2.1	The Poincaré-Bendixson Theorem and Its Applications to Economics .....	72
5.2.2	The Hopf Bifurcation Theorem .....	75
5.3	The Simplified Keynesian Business Cycle Model .....	78
5.4	Non-equilibrium in a Disequilibrium Model .....	81
5.5	Monetary Cycles in the Generalized Tobin Model .....	85
5.6	Oscillations in van der Ploeg's Hybrid Growth Model .....	90
5.7	Periodic Optimal Employment Policy .....	94
5.8	Optimal Economic Growth Associated with Endogenous Fluctuations .....	97
5.9	Remarks on Possible Further Bifurcations from Limit Cycles ..	100
5.10	Competitive Business Cycles in an Overlapping Generations Economy – A Discrete Model .....	102
<b>6.</b>	<b>Economic Chaos in Deterministic Systems .....</b>	<b>106</b>
6.1	Chaos in Deterministic Systems .....	106
6.2	Economic Chaos in a Discrete System .....	108
6.3	Aperiodic Optimal Economic Growth .....	114
6.4	Urban Dynamics – The Lorenz System .....	117
6.5	Chaos in an International Economic Model .....	121
6.6	Chaos and Economic Forecasting .....	122
6.7	Remarks .....	125
	Appendix: Some Criteria for Distinguishing Different Attractors .....	126
A.1	The Lyapunov Exponents for Differential Equations ...	126
A.2	The Lyapunov Exponents for Discrete Maps .....	128
A.3	The Signal, Power Spectrum, Autocorrelation Function and Poincaré Map .....	129
<b>7.</b>	<b>Stochastic Processes and Economic Evolution .....</b>	<b>132</b>
7.1	Random Processes and Economic Evolution .....	132
7.2	Stochastic Processes – An Introduction .....	134
7.2.1	Some Concepts in Probability Theory .....	135
7.2.2	Stochastic Processes .....	137
7.3	Birth-Death Processes and the Master Equation .....	140
7.4	A Non-equilibrium Model of the Schumpeter Clock .....	144
7.5	Effects of Noise on the Nonlinear Stochastic Systems Close to Critical Points .....	152
7.6	Effects of Random Environment on a Two-Dimensional Deterministic System Near Critical Points .....	156
7.7	Conclusions .....	159
<b>8.</b>	<b>Urban Pattern Formation Process – Stability,     Structural Changes and Chaos .....</b>	<b>161</b>
8.1	Continuous Spatial Economics and Description of Urban Pattern Formation .....	161

8.2	The Implications of Structural Stability in the Two-Dimensional Economy .....	165
8.3	Economic Cycles in Puu's Spatial Multiplier- Accelerator Business Model .....	170
8.4	Spatial Diffusional Effects as a Stabilizer .....	174
8.5	Separation and Coexistence of Residents .....	177
8.6	Long-Term Traveling-Wave Urban Pattern .....	182
8.7	Instabilities and Urban Pattern Formation .....	185
	Appendix: Structural Changes in Two Pattern Formation Models .....	185
	A.1    A Model for Morphogenesis .....	186
	A.2    The Brusselator .....	188
<b>9.</b>	<b>The Haken Slaving Principle and Time Scale in Economic Analysis .....</b>	<b>193</b>
9.1	The Haken Slaving Principle .....	193
9.2	The Center Manifold Theorem .....	195
9.3	Singular Perturbations .....	198
9.4	Fast Variable Versus Slow Variable in Economic Analysis .....	202
9.5	The Time Scale in Economic Analysis .....	205
9.6	Another Problem – Understanding a Dynamic Man .....	208
	Appendix: The Slaving Principle for Stochastic Differential Equations	209
<b>10.</b>	<b>Implications of Synergetic Economics .....</b>	<b>213</b>
10.1	Synergetic Economics and Its Relations to Synergetics .....	213
10.2	Relations to Traditional Dynamic Economics .....	214
10.3	Competitive and Planned Economies and Synergetic Economics .....	218
10.4	Implications for Developed and Developing Economies .....	220
10.5	Chance and Necessity in Economic Life .....	222
10.6	Policy Decision in a Chaotic World .....	223
10.7	Relations Between Microeconomics and Macroeconomics .....	225
<b>11.</b>	<b>Conclusions and Prospects for Further Research .....</b>	<b>228</b>
	<b>References .....</b>	<b>231</b>
	<b>Subject Index .....</b>	<b>239</b>