Universitext

Adriaan C. Zaanen

Continuity, Integration and Fourier Theory

Springer-Verlag Berlin Heidelberg New York
London Paris Tokyo

Adriaan C. Zaanen
University of Leiden
Department of Mathematics
P. O. Box 9512
NL-2300 RA Leiden
The Netherlands

Mathematics Subject Classification (1980): 26-XX, 28Axx, 42Axx

ISBN-13: 978-3-540-50017-9 e-ISBN-13: 978-3-642-73885-2
DOI: 10.1007/978-3-642-73885-2

2141/3140-543210 - Printed on acid-free paper

Preface

This book is a textbook for graduate or advanced undergraduate students in mathematics and (or) mathematical physics. It is not primarily aimed, therefore, at specialists (or those who wish to become specialists) in integration theory, Fourier theory and harmonic analysis, although even for these there might be some points of interest in the book (such as for example the simple remarks in Section 15). At many universities the students do not yet get acquainted with Lebesgue integration in their first and second year (or sometimes only with the first principles of integration on the real line). The Lebesgue integral, however, is indispensable for obtaining a familiarity with Fourier series and Fourier transforms on a higher level; more so than by using only the Riemann integral. Therefore, we have included a discussion of integration theory – brief but with complete proofs – for Lebesgue measure in Euclidean space as well as for abstract measures. We give some emphasis to subjects of which an understanding is necessary for the Fourier theory in the later chapters. In view of the emphasis in modern mathematics curricula on abstract subjects (algebraic geometry, algebraic topology, algebraic number theory) on the one hand and computer science on the other, it may be useful to have a textbook available (not too elementary and not too specialized) on the subjects – classical but still important to-day – which are mentioned in the title of this book.

The book consists of four parts. The first part (three chapters) is devoted to a discussion of simple properties of (real or complex) continuous functions on k-dimensional space, followed by the theorems of Korovkin and Stone-Weierstrass. The third chapter contains the elementary theory of Fourier series of continuous functions. The sections on the Stone-Weierstrass theorem can be omitted if desired. In the second part (two chapters) we deal with integration and with L_p-spaces. Convolutions and approximate identities in these spaces receive appropriate attention in view of their importance for what follows. The third part consists of two chapters, one on Fourier series of Lebesgue summable functions and one on the Fourier transform.

The emphasis is on convergence (pointwise or in the sense of Cesaro or Abel summability). The space L_2 receives the special attention it deserves (theorems of Riesz-Fischer and Plancherel). In the fourth part we present several applications. Some of these are of a somewhat more advanced nature, such as those in the section on functions of analytic type and the one on the Hausdorff-Young theorem. The treatment of the heat and wave equations is in the spirit of the discussion in the Dym-McKean book.

The book contains more than sixty exercises, mainly in the second and third parts, some with hints for the solution. At several places, in particular in the sections on Fourier series and Fourier transforms, these exercises form an essential addition to the theory.

Throughout the book familiarity with basic elementary facts in mathematical analysis and linear algebra is assumed.

For further study we refer to the extensive treatises by A. Zygmund (mainly on Fourier series) and R.E. Edwards (Fourier series only), the briefer books by Y. Katznelson and H. Helson (both on an advanced level) and, for those interested in various applications, the fine book by H. Dym and H.P. McKean. The paper by W.A.J.B. Coppel contains an interesting survey of the history of Fourier theory. Finally, the recent book by A. Torchinsky on real-variable methods in harmonic analysis leads the reader to the frontiers of modern research in this area. Details about these books and about the papers mentioned in the text can be found in the list of references at the end of the book.

I take pleasure in thanking my wife Ada for having taken upon her the work of transforming my hand-written pages into a typed manuscript on the basis of which the final camera-ready manuscript was prepared.

Leiden, November 1988 Adriaan C. Zaanen

Contents