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Y. Kuramoto

Chemical Oscillations, Waves, and Turbulence

With 41 Figures

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Professor Dr. Yoshiki Kuramoto

Research Institute for Fundamental Physics, Yukawa Hall, Kyoto University
Kyoto 606, Japan

Series Editor:

Professor Dr. Dr. h. c. Hermann Haken

Institut für Theoretische Physik der Universität Stuttgart, Pfaffenwaldring 57/IV,
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Preface

This book is intended to provide a few asymptotic methods which can be applied to the dynamics of self-oscillating fields of the reaction-diffusion type and of some related systems. Such systems, forming cooperative fields of a large number of interacting similar subunits, are considered as typical synergetic systems. Because each local subunit itself represents an active dynamical system functioning only in far-from-equilibrium situations, the entire system is capable of showing a variety of curious pattern formations and turbulencelike behaviors quite unfamiliar in thermodynamic cooperative fields. I personally believe that the nonlinear dynamics, deterministic or statistical, of fields composed of similar active (i.e., non-equilibrium) elements will form an extremely attractive branch of physics in the near future.

For the study of non-equilibrium cooperative systems, some theoretical guiding principle would be highly desirable. In this connection, this book pushes forward a particular physical viewpoint based on the *slaving principle*. The discovery of this principle in non-equilibrium phase transitions, especially in lasers, was due to Hermann Haken. The great utility of this concept will again be demonstrated in this book for the fields of coupled nonlinear oscillators.

The topics I have selected strongly reflect my personal interest and experiences, so that this book should not be read as a standard textbook. Nevertheless, the spirit by which the present theory is guided may stimulate those students in various fields of science who are fascinated at all by the curiosity of the self-organization in nature.

I am particularly grateful to Professor H. Haken who initially suggested that I write a book on this subject. I wish to thank Dr. H. Lotsch of Springer-Verlag for his patience in waiting for my never-ending manuscript. I am also indebted to Mrs. K. Honda for painstaking typing assistance.

Kyoto, February 1984

Yoshiki Kuramoto

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