Grundlehren der mathematischen Wissenschaften 228

A Series of Comprehensive Studies in Mathematics

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Integrals and Operators

Second Revised and Enlarged Edition

Springer-Verlag
Berlin Heidelberg New York 1978

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AMS Subject Classifications (1970): 22 B xx, 22 C 05, 22 D xx, 28-XX, 43-XX, 46-XX, 47-XX

ISBN-13: 978-3-642-66695-7 e-ISBN-13: 978-3-642-66693-3
DOI: 10.1007/978-3-642-66693-3

Library of Congress Cataloging in Publication Data. Segal, Irving Ezra.
Integrals and operators (Grundlehren der mathematischen Wissenschaften; 228).
Bibliography: p. Includes index. 1. Integrals, Generalized. 2. Functional analysis.
I. Kunze, Ray Alden, 1928—joint author. II. Title. III. Series: Die Grundlehren der mathematischen Wissenschaften in Einzeldarstellungen; 228.
QA312.S4. 1978. 515'.42. 77-16682

Softcover reprint of the hardcover 2nd edition 1978

2141/3140-543210

PREFACE TO THE SECOND EDITION

Since publication of the First Edition several excellent treatments of advanced topics in analysis have appeared. However, the concentration and penetration of these treatises naturally require much in the way of technical preliminaries and new terminology and notation. There consequently remains a need for an introduction to some of these topics which would mesh with the material of the First Edition. Such an introduction could serve to exemplify the material further, while using it to shorten and simplify its presentation.

It seemed particularly important as well as practical to treat briefly but cogently some of the central parts of operator algebra and higher operator theory, as these are presently represented in book form only with a degree of specialization rather beyond the immediate needs or interests of many readers. Semigroup and perturbation theory provide connections with the theory of partial differential equations. C^*-algebras are important in harmonic analysis and the mathematical foundations of quantum mechanics. W^*-algebras (or von Neumann rings) provide an approach to the theory of multiplicity of the spectrum and some simple but key elements of the grammar of analysis, of use in group representation theory and elsewhere. The theory of the trace for operators on Hilbert space is both important in itself and a natural extension of earlier integration-theoretic ideas.

Accordingly, four chapters have been added, one dealing with each of the subjects indicated. These form a logical extension of the standpoint of the First Edition, and at the same time convey the fundamentals of subjects which are central for aspects of higher physical mathematics, group representation theory, and growing applications to analysis on manifolds.

The opportunity has been taken to correct errors, and terminological variations, as well as some expository lapses in the First Edition, which were kindly pointed out to us by conscientious readers. It is hoped the resulting volume will be useful to students and scientists in other fields who may be interested in a cultured overview of modern analysis and its logical structure which retains continuous connections with traditional real variable theory.

PREFACE TO THE FIRST EDITION

This book is intended as a first graduate course in contemporary real analysis. It is focused about integration theory, which we believe is appropriate. For a variety of reasons—in the interests of logic, flexibility, and curricular economy, among others—we have assumed that the reader or student is already familiar with the rudiments of modern mathematics (by this we mean the most elementary aspects of set theory, general topology, and algebra, as well as some exposure to rigorous analysis). These are not so much technical requirements—although basic concepts such as set, topological space, and uniform convergence are taken entirely for granted—as requirements of mathematical maturity and of understanding of the elementary grammar and language of modern mathematics. Assuming the adequate mathematical "aging" of the student, the book is quite self-contained. Results such as the Stone-Weierstrass theorem, the existence of a partition of unity, etc., are given full proofs rather than disposed of by reference to hypothesized preliminaries.

The aim of the book is primarily cultural, rather than vocational; the authors strive to expose the student to modern analytical thought and if possible to train him to think in such terms rather than to load him with all available information on the subject. Nevertheless, the book should represent a proper introduction to real analysis for students intending to concentrate in analysis, as well as a (possibly terminal) general course in the subject for those with other scientific interests. Indeed, thought cannot take place in a vacuum, and contrived illustrations of the theory have a way of turning out not to be as truly representative or interesting as illustrations of the actual usage of the theory for vital mathematical purposes. For this reason the book has been built around material of maximal current mathematical importance and depth, a technical mastery of which should go hand in hand with an appreciation of the general ideas.

The book is a revision, adaptation, and extension of lecture notes of courses in Integration Theory given at the University of Chicago and in Real Variable Theory at Massachusetts Institute of Technology by the first-named author. The first half of the book is suitable for a one-semester course in Lebesgue integration theory, including both abstract and classical real variable aspects. Its heart is a fresh presentation of the Daniell approach, which, combined with the use of general topological ideas, attains a high level of generality and completeness without burdening the student with heavy machinery or bulky technicalities. This material should provide a cultural experience for the student comparable to his first exposure to the calculus; indeed, the success of this theory against what appear initially as overwhelming scientific odds, and its broad applicability, render it one of the comparable intellectual achievements of mathematics. Many examples and a considerable variety of exercises, at all levels of difficulty, serve to illustrate the theory and to indicate the continuous transition between the concrete and abstract phases of integration theory. Theoretical ramifications which are secondary from the standpoint of the overall theoretical development, although frequently of considerable importance, are included among the more difficult exercises; the student is assisted with hints, and the arrangement is designed to encourage learning by students' investigations under the guidance of the instructor or by self-discovery. The exercises range in difficulty from easy ones which simply confirm an understanding of the text to relatively difficult ones, distinguished by a *, which in a controlled way, introduce the student to the beginnings of research. The * is also used to distinguish material (several sections and one chapter) which may be omitted without disturbing the main line of development.

The book as a whole is quite unified. Integration theory provides the main examples for the treatment of linear topological spaces and their duality. In the more structured situations provided by groups of transformations, new aspects of function theory arise from the consideration of invariant measures. The reducibility of commutative spectral analysis in Hilbert space to integration theory makes it natural, as well as economical, to develop spectral theory from this viewpoint. For these reasons the book may be used for the second half of a one-year course in Real Analysis, which merges naturally with the first half, provides basically new material of general importance, and yet serves at the same time to build on and provide a capstone for the student's earlier exposure to linear algebra and integration. The completion of such a course should provide the student with the key real-analytical background for work in other parts of modern mathematics; for more advanced work in analysis, whether of a more abstract or concrete variety; and for contemporary theoretical physics. We especially feel that the book takes the student rather quickly, but not too abruptly, to a good jumping-off place for the study of Fourier analysis, linear partial differential operators, the theory of group representations, operator algebras, and abstract probability theory.

Although there are now many quite competent treatments in textbook form of Lebesgue integration theory, as well as some on introductory functional analysis, we feel that none of these books achieves quite what this one is intended to do. With our treatment, it is possible to take the suitably prepared student in one year at a properly measured pace through basic contemporary real analysis, giving him the feel of the subject, a clear indication of its sweep, and an adequately detailed mastery of a number of central features. Our general viewpoint is partially in the direction of an earlier exposition by one of us on algebraic integration theory, i.e., toward the utilization of abstract integrative ideas (cf. References, p. 365); this has always been one of the long-term trends in mathematics, enforcing a type of consolidation which may be essential to prevent undue scientific complexity and bulk from imposing a crushing burden on the development of fresh ideas and methods. At the same time, as already indicated, the continuous linkage between the abstract theory and the concrete analytical situation has been everywhere insisted on—in the motivational material, in the examples, and in the exercises. We believe that this book is more likely to help cure "abstractionitis"—an unfortunate but not uncommon side effect of otherwise highly beneficial inoculations with modern mathematical ideas—than to cause it. We have made some technical innovations where they appeared useful to serve our central ideas, but have avoided them otherwise. Thus the treatment of "large" measure spaces is curtailed in the text, such primarily technical developments being outlined in the exercises; on the other hand, the uniform-space approach to the construction of invariant measures has been adopted.

As it has turned out, the book is fairly flexible from an instructional viewpoint. An independent short course, in the nature of an introduction to functional analysis and spectral theory in Hilbert space, may be given on the basis of the second half of the book, exclusive of Chapter 7, for students already familiar with the abstract Lebesgue integral.

We are much indebted to many colleagues and students for general advice and specific comments and for reducing the number of errors in the manuscript to what we hope is a superficial level. In particular, lengthy lists of corrections to draft manuscripts were supplied by Robert Kallman, Michael Weinless, and Alan Weinstein.

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