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Abstract Harmonic Analysis

Volume II

Structure and Analysis for Compact Groups
Analysis on Locally Compact Abelian Groups



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*Dem Andenken
John von Neumanns
gewidmet*

Preface

This book is a continuation of Volume I of the same title [Grundlehren der mathematischen Wissenschaften, Band 115]. We constantly cite definitions and results from Volume I.¹ The textbook *Real and abstract analysis* by E. HEWITT and K. R. STROMBERG [Berlin · Göttingen · Heidelberg: Springer-Verlag 1965], which appeared between the publication of the two volumes of this work, contains many standard facts from analysis. We use this book as a convenient reference for such facts, and denote it in the text by RAAA. Most readers will have only occasional need actually to read in RAAA.

Our goal in this volume is to present the most important parts of harmonic analysis on compact groups and on locally compact Abelian groups. We deal with general locally compact groups only where they are the natural setting for what we are considering, or where one or another group provides a useful counterexample. Readers who are interested only in compact groups may read as follows: § 27, Appendix D, §§ 28–30 [omitting subheads (30.6)–(30.60) if desired], (31.22)–(31.25), §§ 32, 34–38, 44. Readers who are interested only in locally compact Abelian groups may read as follows: §§ 31–33, 39–42, selected Miscellaneous Theorems and Examples in §§ 34–38. For all readers, § 43 is interesting but optional.

Obviously we have not been able to cover all of harmonic analysis. The field, already immense, is growing rapidly at the present day. We were limited by space, by time, by our own abilities. We have presented the parts of the subject that every harmonic analyst must know: representations of compact groups; the WEYL-PETER theorem; PLANCHEREL'S theorem; WIENER'S Tauberian theorem. Beyond this, we have been guided largely by personal inclination. As the writing progressed, one question led naturally to another.

We have omitted special topics that are not needed for our main goals and that are treated in other monographs: RUDIN [10]; R. E. EDWARDS [7], [10]; KATZNELSON [3]; KAHANE and SALEM [3]; MAURIN [1]. We regret not having presented any Lie theory beyond the rudimentary facts set down in § 29. Plainly a detailed description of the continuous unitary irreducible representations of the classical compact

¹ An exception is the Bibliography: every work cited in Vol. II is listed at the end of Vol. II.

groups, going beyond *e.g.* BOERNER [1], and of the decompositions of their tensor products, would be of immense value for noncommutative harmonic analysis. The time seems not yet ripe for such an enterprise. A larger omission is our failure to study the algebra $\mathcal{M}(G)$ [G a compact or locally compact Abelian group]. This algebra presents many riddles, but enough is known for a full-scale treatment to be appropriate.

We could not have written this book without help. We are deeply grateful for the generous assistance offered by our friends. Valuable advice has been received from ROBERT B. BURCKEL, CLIFFORD V. COMISKY, RAOUF DOSS, ROBERT E. EDWARDS, LEE W. ERLEBACH, J. M. G. FELL, FRANZ VON KRBEK, A. JEANNE LADUKE, HORST LEPTIN, GEORGE W. MACKEY, JOHN R. McMULLEN, WILLARD A. PARKER, RICHARD S. PIERCE, ROGER W. RICHARDSON, KARL R. STROMBERG, THOMAS A. SWANSON, EUGENE P. WIGNER, and JOHN H. WILLIAMSON. Significant contributions to the final form of the monograph were made by RICHARD ILTIS, BARRY E. JOHNSON, and DANIEL RIDER. Our special thanks are due to HERBERT S. ZUCKERMAN, who has helped us far more than anyone else.

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