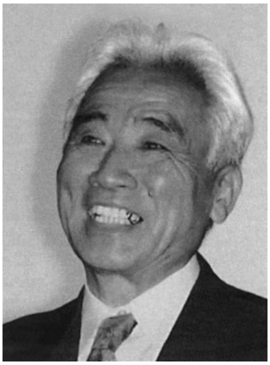
Classics in Mathematics

Shôichirô Sakai C*-Algebras and W*-Algebras



Shôichirô Sakai was born in 1928 in Kanuma, Japan. He received his B. A. in mathematics in 1953 and his doctorate in 1961 from Tohoku University, Sendai. He was a research assistant there (1953–1960), then a faculty member of Waseda University, Tokyo (1960–1964). After 2 years on the faculty of the University of Pennsylvania, he became a professor in 1966 and stayed there till 1979 when he returned to Japan to a chair at Nihon University, Tokyo. Prof. Sakai has also held visiting positions at Yale University (1962–64) and at MIT (1967–68). His main interests are in operator algebras, functional analysis and mathematical physics.

Shôichirô Sakai

C*-Algebras and W*-Algebras

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To Masato and Kiyoshi

Preface

The theory of operator algebras in a Hilbert space was initiated by von Neumann [126] in 1929. In the introduction to the paper [119] in 1936, Murray and von Neumann stated that the theory seems to be important for the formal calculus with operator-rings, the unitary representation theory of groups, a quantum mechanical formalism and abstract ring theory. These predictions have been completely verified. Furthermore the theory of operator algebras is now becoming a common tool in a number of fields of mathematics and theoretical physics beyond those mentioned by Murray and von Neumann. This is perhaps to be expected, since an operator algebra is an especially well-behaved infinite-dimensional generalization of a matrix algebra. Therefore one can confidently predict that the active involvement of operator algebra theory in various fields of mathematics and theoretical physics will continue for a long time.

Another application of the theory is to the study of a single operator in a Hilbert space (see, for example, [99]). Nowadays we may add further the theory of singular integral operators and *K*-theory as other applications. Such diversifications of the theory of operator algebras have already made a unified text book concerning the theory virtually impossible. Therefore I have no intention of giving a complete coverage of the subject. I will rather take a somewhat personal stand on the selection of material—i. e., the selection is concentrated heavily on the topics with which I have been more or less concerned (needless to say, there are many other very important contributors to those topics. The reader will find the names of authors who have made remarkable contributions to those discussed in the concluding remarks of each section). Consequently parts of the book tend to be somewhat monographic in character.

Let me explain briefly about the contents. There are essentially two different ways of studying the operator *-algebras in Hilbert spaces. The first alternative is to assume that the algebra is weakly closed (called a W^*-algebra). These algebras are also called Rings of operators, and more recently, von Neumann algebras.

The earliest study along this line is due to von Neumann in 1929. In a series of five memoirs beginning with [119], Murray and von Neumann laid the foundation for the theory of W^*-algebras. Virtually all of the later work on these algebras is based directly or indirectly on their pioneering work.

We call a W^*-algebra a factor if its center is just the complex numbers. Murray and von Neumann concentrated most of their attention on factors. However, von Neumann [132] obtained a reduction theory by which the study of a general W^*-algebra may be to a large extent reduced to the case of a factor. At the same time a number of authors have pushed through the major portions of a global theory for general W^*-algebras (the reader may find a long list of papers by many authors in the bibliography in [37]).

The second alternative is to assume only that the algebra is uniformly closed (called a C^*-algebra). The earliest study along this line is due to Gelfand and Naimark [55] in 1943. A notable advantage of the C^*-algebra is the existence of an elegant system of intrinsic postulates, formulated by Gelfand and Naimark, which gives an abstract characterization of these algebras.

Using this approach, Segal [180] in 1947 initiated a study of C^*-algebras. The subsequent development which contains many beautiful results (cf. Chapters 1, 3, & 4) has been carried out by a number of authors.

The theory of C^*-algebras fits naturally into the theory of Banach algebras, and in certain respects they are among the best behaved examples of infinite dimensional Banach algebras.

In chapter 1, the characterization of W^*-algebras obtained in [149] is used to define W^*-algebras as abstract Banach algebras, like C^*-algebras, and to develop the abstract treatments of both W^*- and C^*-algebras.

Chapter 2 is concerned mainly with the classification and representation theory of W^*-algebras which are developed along classical standard lines. Some proofs may be new.

In chapter 3, the reduction theory is discussed. Here, a modern method, developed recently in [146], [159], [218] (i.e. the decomposition theory of states) is used. Also discussed are some recent results obtained by theoretical physicists.

Chapter 4 consists of some special topics from the theories of W^*-algebras and C^*-algebras. This chapter is the most personal in the book. All the topics covered are ones with which I have been more or less concerned. They are: Derivations and automorphisms on operator algebras; examples of factors; examples of non-trivial global W^*-algebras; type I C^*-algebras and a Stone-Weierstrass theorem for C^*-algebras.

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Philadelphia, Pa. U.S.A., January 1971 S. Sakai

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