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A Series of Comprehensive Studies in Mathematics

Bernard Maskit

Kleinian Groups

With 67 Figures

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Introduction

The modern theory of Kleinian groups starts with the work of Lars Ahlfors and Lipman Bers; specifically with Ahlfors' finiteness theorem, and Bers' observation that their joint work on the Beltrami equation has deep implications for the theory of Kleinian groups and their deformations. From the point of view of uniformizations of Riemann surfaces, Bers' observation has the consequence that the question of understanding the different uniformizations of a finite Riemann surface poses a purely topological problem; it is independent of the conformal structure on the surface. The last two chapters here give a topological description of the set of all (geometrically finite) uniformizations of finite Riemann surfaces. We carefully skirt Ahlfors' finiteness theorem. For groups which uniformize a finite Riemann surface; that is, groups with an invariant component, one can either start with the assumption that the group is finitely generated, and then use the finiteness theorem to conclude that the group represents only finitely many finite Riemann surfaces, or, as we do here, one can start with the assumption that, in the invariant component, the group represents a finite Riemann surface, and then, using essentially topological techniques, reach the same conclusion.

More recently, Bill Thurston wrought a revolution in the field by showing that one could analyze Kleinian groups using 3-dimensional hyperbolic geometry, and there is now an active school of research using these methods. The work here shares some foundation with Thurston's methods, but an exploration of his deep and beautiful results lies beyond the scope of this book. Some of the basic material developed here in Chapters II, and IV–VII is also useful as foundation for Thurston's work.

This book was designed to be usable as a textbook for a one year advanced graduate course in Kleinian groups. Except for Chapters III and VIII, one could follow the material in the order given for such a course. For the most part, Chapter III is included as a reference for some more or less well-known, relatively easy to derive, but hard to find facts about regular coverings of surfaces. Chapter VIII is a collection of examples of Kleinian groups with diverse properties. The examples range from easy illustrations in the use of combination theorems, to fairly complicated constructions of groups with esoteric properties.

For someone first learning about Kleinian groups, there are many difficulties in the theory caused by the presence of parabolic or elliptic elements in the group.

On first reading, one should take the point of view that one is only interested in purely loxodromic geometrically finite Kleinian groups. When read from this point of view, Chapters V and IX almost disappear, and Chapters VI, VII, VIII, and X become significantly shorter. After the general theory for purely loxodromic groups is clear, then one can go back as necessary and fill in the difficulties and complications caused by the presence of elliptic and parabolic elements.

The basic organization of the book has three levels. The highest level are the chapters; these are labelled I through X, and of course they also have names. The next level down are the sections, which are labelled A, B, C, ..., and these also have names. The lowest level consists of subsections, labelled 1, 2, 3, ... Some of the subsections in Chapter VIII have names, the others do not. Each subsection contains at most one statement of a theorem, corollary, proposition, or lemma, so internal references usually do not use the words theorem, lemma, etc. but merely refer to the appropriate subsection. For example, from within Chapter VII, a reference to the theorem whose statement appears in Chapter VII, section C, subsection 2 is given as simply "C.2.". A reference to the same theorem from outside Chapter VII is given as "VII.C.2.".

The figures are numbered separately. These are referred to, for example, as Fig. VIII.E.8; this is the 8th figure is section E of Chapter VIII; it need not have anything to do with subsection 8 of VIII.E. Similarly, the few formulas are also separately numbered.

The exercises at the end of each chapter were put in for the usual reasons, and are quite uneven in terms of difficulty (this is not to say that any of them are deliberately unsolved or unsolvable). In broad outline, they progress according to the material in each chapter, but there are also some problems that were added on at the end.

This book had its origins in 1970, when, as a Sloan Foundation Fellow at the University of Warwick, I started to write a set of notes. Since then I taught a course in Kleinian groups several times, and slowly expanded the notes until they grew large enough to become seemingly unmanageable. Fortunately, at just about that time, personal computers came into being, so I bought one, bought a technical word processor, and set about rewriting and revising my manuscript. I also underwent some changes in my personal life, which may have something to do with my increased ability to organize myself and my notes.

During the course of the years I was writing and organizing this book, I profited immensely from the encouragement and advice of many friends and colleagues. I also would never have finished this book if it were not for my wife, Wilma, whose loving patience and quiet support have been a great source of strength. I owe a deep debt of gratitude to my teacher, Lipman Bers, who both taught me what mathematical research was all about, and, through all these years, has been a constant source of encouragement and sound advice. I also wish to thank my long-standing friend and colleague, Irwin Kra, who always has a kind word, both for his strong encouragement, and for his help in reading some of the preliminary drafts. I had help from many people, who pointed out errors in drafts, and helped proofread; in this connection, I especially wish to thank Bill Abikoff, Jim Anderson, Ara Basmajian, Andy Haas, Blaise Heltai, Peter Matelski, and Perry Susskind. There are many others as well, who pointed out errors or made suggestions; I thank them all, and I hope I have not slighted anyone by failing to mention his or her name. Of course, there are still errors remaining, hopefully none of them serious; it seems to be a general principle that no matter how many times one goes through the manuscript, one always finds more errors. Thanks are also due to Werner Fenchel for teaching me about half-turns; the contents of section V.B are essentially due to him. I also had help from Gilbert Baumslag with the example of a locally free group. Finally, thanks are due to the National Science Foundation for support during the many years this book was in preparation.

Stony Brook, July 1987 Bernard Maskit

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