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The Mathematics of Paul Erdős I



Springer

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IN MEMORIAM

PAUL ERDŐS

26. 3. 1913 – 20. 9. 1996

The week before these volumes were scheduled to go to press, we learned that Paul Erdős died on September 20, 1996. He was 83. Paul died while attending a conference in Warsaw, on his way to another meeting. In this respect, this is the way he wanted to “leave”. In fact, the list of his last month’s activities alone inspires envy in much younger people.

Paul was present when the completion of this project was celebrated by an elegant dinner in Budapest for some of the authors, editors and Springer representatives attending the European Mathematical Congress. He was especially pleased to see the first copies of these volumes and was perhaps surprised (as were the editors) by the actual size and impact of the collection. We hope that these volumes will provide a source of inspiration as well as a last tribute to one of the great mathematicians of our time. And because of the unique lifestyle of Paul Erdős, a style which did not distinguish between life and mathematics, this is perhaps a unique document of our times as well.

R. L. G.
J. N.

Preface

In 1992, when Paul Erdős was awarded a Doctor Honoris Causa by Charles University in Prague, a small conference was held, bringing together a distinguished group of researchers with interests spanning a variety of fields related to Erdős' own work. At that gathering, the idea occurred to several of us that it might be quite appropriate at this point in Erdős' career to solicit a collection of articles illustrating various aspects of Erdős' mathematical life and work. The response to our solicitation was immediate and overwhelming, and these volumes are the result.

Regarding the organization, we found it convenient to arrange the papers into six chapters, each mirroring Erdős' holistic approach to mathematics. Our goal was not merely a (random) collection of papers but rather a thoroughly edited volume composed in large part by articles explicitly solicited to illustrate interesting aspects of Erdős and his life and work. Each chapter includes an introduction which often presents a sample of related Erdős' problems "in his own words". All these (sometimes lengthy) introductions were written jointly by editors.

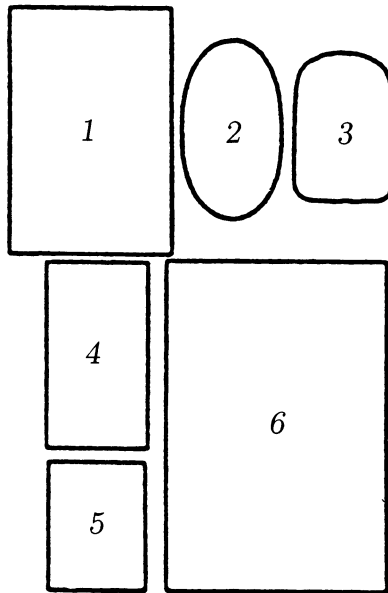
We wish to thank the nearly 70 contributors for their outstanding efforts (and their patience). In particular, we are grateful to Béla Bollobás for his extensive documentation of Paul Erdős' early years and mathematical high points (in the first part of this volume); our other authors are acknowledged in their respective chapters. We also want to thank A. Bondy, G. Hahn, I. Ouhel, K. Marx, J. Načeradský and Ché Graham for their help and for the use of their works. At various stages of the project, the book was supported by AT&T Bell Laboratories, GAČR 2167 and GAUK 351. We also are indebted to Dr. Joachim Heinze and Springer Verlag for their encouragement and support. Finally, we would like to record our extreme debt to Susan Pope (at AT&T Bell Laboratories) who somehow (miraculously) managed to convert more than 50 manuscripts of all types into the attractive form they now have.

Here then is a unique portrait of a man who has devoted his whole being to "proving and conjecturing" and to the pursuit of mathematical knowledge and understanding. We hope that this will form a lasting tribute to one of the great mathematicians of our time.

R. L. Graham
J. Nešetřil

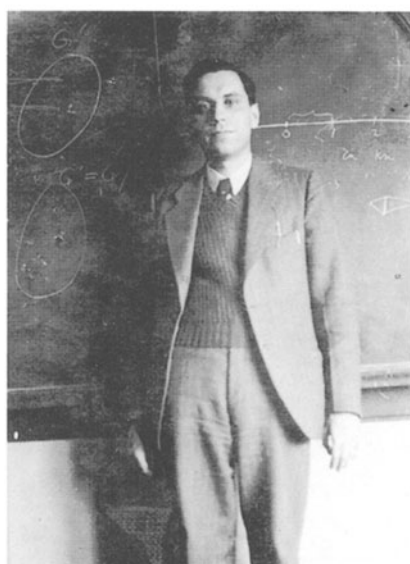
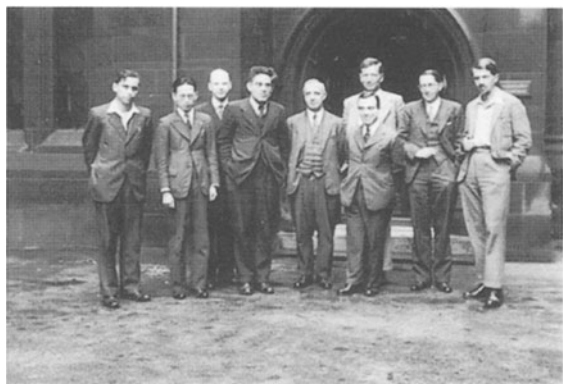
VI Illustrations

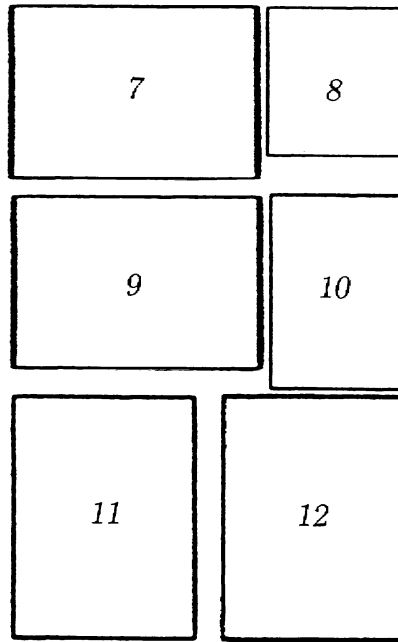




- 1 With mother at lake Balaton 1916-1917
2-5 School and highschool in Budapest;
Passport photos
6 About 1921
(a favourite picture of his mother)

VIII Illustrations





- 7 Manchester number theory group in 1937 or 1938.
 From left to right: Erdős, Chao Ko, Zsilinskas,
 Mordell, Bailey, Mahler, Heilbronn, Davenport, Duval
- 8 P. Erdős and R. Rado (early 50's)
- 9 P. Erdős and L. Moser 1957 (Edmonton)
- 10 and 12 50's
- 11 P. Erdős and Dorothy Maharam Stone,
 Princeton 1940

Contents of The Mathematics of Paul Erdős I

Paul Erdős — Life and Work	1
B. BOLLOBÁS	
I. Early Days	43
Introduction	45
Some of My Favorite Problems and Results	47
P. ERDŐS	
Encounters with Paul Erdős	68
A. H. STONE	
Did Erdős Save Western Civilization?	74
C. A. B. SMITH	
Integers Uniquely Represented by Certain Ternary Forms	86
I. KAPLANSKY	
On Cubic Graphs of Girth at Least Five	95
W. T. TUTTE	
II. Number Theory	99
Introduction	101
Classical Results on Primitive and Recent Results on Cross-Primitive Sequences	104
R. AHLWEDE AND L. H. KHACHATRIAN	
Sur la Non-dérivabilité de Fonctions Périodiques Associées à Certaines Formules Sommatoires	117
G. TENENBAUM	
On Additive Representation Functions	129
A. SÁRKÖZY AND V. T. SÓS	

Arithmetical Properties of Polynomials	151
A. SCHINZEL	
Cross-Disjoint Pairs of Clouds in the Interval Lattice	155
R. AHLWEDE AND N. CAI	
Dense Difference Sets and their Combinatorial Structure	165
V. BERGELSON, P. ERDŐS, N. HINDMAN AND T. LUCZAK	
On Primes Recognizable in Deterministic Polynomial Time	176
S. KONYAGIN AND C. POMERANCE	
Ballot Nummmbers, Alternating Products, and the Erdős- Heilbronn Conjecture	199
M. B. NATHANSON	
Integer Sets Containing no Solution to $x + y = 3z$	218
F. R. K. CHUNG AND J. L. GOLDWASSER	
On Landau's Function $g(n)$	228
J.-L. NICOLAS	
On Divisibility Properties of Sequences of Integers	241
A. SÁRKÖZY	
Some Methods of Erdős Applied to Finite Arithmetic Progressions	251
T. N. SHOREY AND R. TIJDEMAN	
1105: First Steps in a Mysterious Quest	268
G. TENENBAUM	
III. Randomness and Applications	277
Introduction	279
Games, Randomness and Algorithms	280
J. BECK	
The Origins of the Theory of Random Graphs	311
M. KAROŃSKI AND A. RUCIŃSKI	
The Erdős Existence Argument	337
J. SPENCER	
On Some Hypergraph Problems of Paul Erdős and the Asymptotics of Matchings, Covers and Colorings	345
J. KAHN	

How Abelian is a Finite Group?	372
L. PYBER	
On Small Size Approximation Models	385
A. A. RAZBOROV	
An Upper Bound for a Communication Game Related to Time-Space Tradeoffs	393
P. PUDLÁK AND J. SGALL	

Contents of The Mathematics of Paul Erdős II

IV. Combinatorics and Graph Theory	1
Introduction	3
Problems in Graph Theory from Memphis	7
R. J. FAUDREE, C. C. ROUSSEAU AND R. H. SCHELP	
Neighborly Families of Boxes and Bipartite Coverings	27
N. ALON	
Cycles and Paths in Triangle-Free Graphs	32
S. BRANDT	
Reconstruction Problems for Digraphs	43
M. AIGNER AND E. TRIESCH	
The Dimension of Random Graph Orders	51
B. BOLLOBÁS AND G. BRIGHTWELL	
Hereditary and Monotone Properties of Graphs	70
B. BOLLOBÁS AND A. THOMASON	
Properties of Graded Posets Preserved by Some Operations	79
S. BEZRUKOV AND K. ENGEL	
Intersection Representations of the Complete Bipartite Graph ...	86
Z. FÜREDI	
Reflections on a Problem of Erdős and Hajnal	93
A. GYÁRFÁS	
The Chromatic Number of the Two-packing of a Forest	99
H. WANG AND N. SAUER	
On the Isolation of a Common Secret	121
D. BEAVER, S. HABER AND P. WINKLER	
Some Remarks on the Cycle Plus Triangles Problem	136
H. FLEISCHNER AND M. STIEBITZ	

V. Ramsey and Extremal Theory	143
Introduction	145
Paul Erdős' Influence on Extremal Graph Theory	148
M. SIMONOVITS	
Ramsey Theory in the Work of Paul Erdős	193
R. L. GRAHAM AND J. NEŠETŘIL	
Memories on Shadows and Shadows of Memories	210
G. O. H. KATONA	
Applications of the Probabilistic Method to Partially Ordered Sets	214
W. T. TROTTER	
A Bound of the Cardinality of Families not Containing Δ -Systems	229
A. V. KOSTOCHKA	
Arrangeability and Clique Subdivisions	236
V. RÖDL AND R. THOMAS	
A Finite Partition Theorem with Double Exponential Bound ...	240
S. SHELAH	
 VI. Geometry	 247
Introduction	249
Extension of Functional Equations	251
J. ACZÉL AND L. LOSONCZI	
Remarks on Penrose Tilings	264
N. G. DE BRUIJN	
Distances in Convex Polygons	284
P. FISHBURN	
The Number of Homothetic Subsets	294
M. LACZKOVICH AND I. Z. RUZSA	
On Lipschitz Mappings onto a Square	303
J. MATOUŠEK	
A Remark on Transversal Numbers	310
J. PACH	
In Praise of the Gram Matrix	318
M. ROSENFELD	

On Mutually Avoiding Sets	324
P. VALTR	
VII. Infinity	329
Introduction	331
The Random Graph	333
P. J. CAMERON	
Paul Erdős' Set Theory	352
A. HAJNAL	
A Few Remarks on a Conjecture of Erdős on the Infinite Version of Menger's Theorem	394
R. AHARONI	
On Order-Perfect Lattices	409
I. KRÍŽ	
The PCF Theorem Revisited	420
S. SHELAH	
Set Theory: Geometric and Real	460
P. KOMJÁTH	
Paul Erdős: The Master of Collaboration	467
J. W. GROSSMAN	
List of Publications of Paul Erdős	477
Postscript	575