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Isaak M. Tsidilkovski

# Electron Spectrum of Gapless Semiconductors

With 58 Figures and 5 Tables



Springer

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## Preface

The lectures given by me as the Max-Planck Professor at the Humboldt University (Berlin) are the roots of this monograph. The course of lectures consisted of two parts. The first part described the methods used to calculate the electron energy spectra of solids, and the second part dealt with the features peculiar to the electron spectra of gapless semiconductors. In this book, the first half of the lectures (spectrum computational methods) is covered in Chap.1. The other half of the lectures is presented in Chaps.2-4, which are dedicated, respectively, to the band structure of ideal (impurity-free) gapless semiconductors, impurity states in gapless semiconductors, and gapless semimagnetic semiconductors. The book treats most problems more comprehensively than was done in the lectures. Additionally, it includes a section dealing with a new gapless semi-magnetic semiconductor: mercury-iron-selenium. The presence of the donor resonance level of iron makes the compound's physical properties so unusual that I could not resist the temptation to describe them in the section concerning semimagnetic semiconductors. Some very recent results are sketched in Chap.5 to demonstrate how the subject of this book is developing.

This monograph gives primary consideration to qualitative interpretations of the problems. Computations are frequently omitted and only the resulting formulas presented for the analysis of the properties and phenomena under discussion. The reader should be familiar with quantum mechanics and statistical physics at the level of a university course.

I would like to thank Profs. R. Keiper, R. Enderlein, K. Herrmann and V. Ebeling for their hospitality and assistance during my stay in Berlin.

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Ekaterinburg  
August 1996

*I.M. Tsidilkovski*

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