

Texts and Monographs in Physics

Series Editors: R. Balian W. Beiglböck H. Grosse E. H. Lieb
N. Reshetikhin H. Spohn W. Thirring

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Maciej Błaszak

Multi-Hamiltonian Theory of Dynamical Systems

With 9 Figures



Springer

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To my wife Elizabeth

Preface

The book aims to provide a comprehensive and easy presentation of modern algebraic theory of integrable nonlinear dynamical systems. This relatively new field of mathematical physics, originating from the theory of solitons, has been intensely developed in the last two decades. Particular attention in this book has been paid to modern multi-Hamiltonian formalism on manifolds and associative Lie algebras, which seem very powerful tools in the investigation of integrable nonlinear systems of different kinds, e.g. field, lattice and mechanical ones.

The book is addressed to graduates of physics and applied mathematics who have elementary background in classical mechanics and differential geometry. My intention was to make the book as easy to read as possible, so many considerations are explained from scratch, all calculation formalisms are explained in detail, almost all theorems and lemmas are proved, and each chapter is supplemented with numerous examples.

I am much indebted to all my co-workers but in particular to two of them. The first person is Professor Benno Fuchssteiner from Paderborn University, who inspired my interest in algebraic methods of soliton theory and who implanted in me his passion for the work in this field. The other person whom I must mention is Professor Stefan Rauch-Wojciechowski from the Linköping University, with whom I have been working for many years and who shared with me many original ideas on algebraic theory of integrable finite-dimensional systems.

Finally, I wish to thank Professor Wolf Beiglböck from Springer-Verlag for his enthusiastic encouragement to write this book, which otherwise might not have been written.

Poznań, April 1998

Maciej Błaszak

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