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B. N. Pshenichnyj

The Linearization Method for Constrained Optimization

Translated from the Russian
by Stephen S. Wilson

With 6 tables

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Boris N. Pshenichnyj
Institute of Cybernetics of the
Academy of the Ukrainian SSR
142/144-4 Oletiya Oktyabrya ul.
25227 Kiev - 207
Ukraina, CIS

Translator
Stephen S. Wilson
Fourwinds
79 New Barn Lane
Prestbury
Cheltenham
Gloucestershire GL 52 3LE
United Kingdom

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Foreword

It is customary in a foreword to write about the practical importance of the problem and to describe the contents of the book. However, the practical importance of solving optimization problems has been unquestioned for some time. A wealth of scientific and popular scientific literature is devoted to this subject, so that it is almost unnecessary to repeat yet again that the theory and techniques of optimization are applied in many problems in economics, automatic control, engineering etc. On the other hand, a comprehensive summary of the contents of the book is given in the first introductory section. Therefore, I shall not dwell on this here. Instead, I shall allow myself a number of general remarks on the theory and the numerical techniques of optimization and the interlinking of these in the solution of a complicated real problem.

Electronic computers were first applied to solve optimization problems in their earliest days. The first such applications involved linear programming problems with a very simple structure, which could be solved by regular methods, and comparatively uncomplicated nonlinear problems, which were solved based on simple inductive considerations and using the facility for vast numbers of numerical computations provided by the computer technology. But the challenge of more and more new problems of increasing size and nonlinear complexity meant that it was no longer possible to use simple techniques and raw power alone. The solution of substantial nonlinear problems required a deeper theoretical study of methods for solving them together with delicate and complicated techniques for obtaining numerical results for each specific problem within a reasonable time. The process of generating such an arsenal of techniques for solving optimization problems has been intently pursued over the last twenty years.

The linearization method, which is the subject of this book, is one of the many fruits of this process.

In fact, as will be seen from what follows, the linearization method is closely related to Newton's method for solving systems of linear equations, to penalty function methods and in particular, to methods involving nonsmooth penalty functions, and, in connection with the latter, to methods of nondifferentiable optimization. The linearization method cannot be successfully applied without the efficient solution of quadratic programming problems; this leads to a connection with conjugate gradient methods and variable metrics. The desire to solve large-scale problems requires the application of techniques developed in linear programming, i.e. techniques for working with sparse matrices, a multiplicative representation of inverse matrices etc.

Finally, it is impossible to study areas and rates of convergence without applying the theory of necessary conditions for extrema, Lagrange multipliers and functions, and the concept of the dual problem. It follows that a comprehensive study of the properties of the linearization method is impossible without applying a wide range of concepts ranging from abstract theory to specifics of the computer-based implementation. Naturally, all this is reflected, to a greater or lesser extent, in the book. These questions cannot not all receive the same attention if the book is to be held to a reasonable size. However, while some problems are only considered superficially (for example, conjugate gradient and variable metric methods, methods for working with sparse matrices), this does not mean that they are unimportant. The contrary is often true, as the example of sparse matrices shows. Were it not for the use of sparse matrices, the time taken for large problems would increase catastrophically.

There is yet another very important reason why specialists who are primarily interested in a specific application of the method should nevertheless be familiar with the associated concepts and ideas. The fact is that the method is based on the solution of a general nonlinear programming problem and thus considerable time may be expended on solving subproblems of this problem. A specific problem (or class of problems) always has specific properties (for example, most constraints have a very simple form) and taking these properties into account by changing certain modules of the algorithm may lead to a considerable reduction in the time taken and the computer storage used. Clearly, such changes cannot be successfully implemented without an understanding of the principal mechanisms which lead to convergence of the algorithm.

Although this book was dictated by an urge to undertake some form of research relating to the linearization method, the author is confident that it will not be the last word on this topic. This is confirmed by the ever increasing number of papers on this and related subjects. Moreover, there are many problems which require deeper and more detailed study. In particular, these include the choice of a rule for transition from the simple to the accelerated linearization method, more precise definition of rules for choosing the step in such a transition, algorithms to change the constants in penalty functions, and techniques for approximating the matrix of second derivatives of Lagrange functions. More detailed consideration should be given to the specific properties of the linearization method when applied to large-scale problems, methods for decomposing the original and the auxiliary problems etc. All these problems are nontrivial, and the author will be pleased if this book serves not only to expand the sphere of practical application of this method but also as a starting point for further improvements and developments.

B.N. Pshenichnyj

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