Texts and Monographs in Physics

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R. Balian W. Beiglböck H. Grosse E. H. Lieb H. Spohn W. Thirring

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Finite Quantum Electrodynamics

The Causal Approach

Second Edition
With 17 Figures

Springer

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Preface

Quantum field theory as it is usually formulated is full of problems with ultraviolet and infrared divergences. This is somewhat surprising, because there is a simple way out which one learns in mathematics. One must only adopt the following two rules. First, use well-defined quantities only, for example free fields. Second, make justified operations only in the calculations; in particular do not multiply certain distributions by discontinuous step functions. If one really follows these rules, then no infinity can appear and life is beautiful. The question then is how to construct the standard theory according to these rules. This one can learn from an old paper by Epstein and Glaser (*Annales de l'Institut Poincaré A 19, p. 211 (1973)*). The main tool in this method is causality.

The causal method was developed by Stückelberg and Bogoliubov in the 1950s. One reason for the limited resonance it found was perhaps the highly non-trivial nature of the causality condition. We therefore start slowly. After a chapter on the classical Dirac theory of electrons and positrons and the quantization of free fields, we study the external field problem in some detail. We will find that the (second quantized) scattering matrix (S-matrix) for this problem is uniquely determined up to a phase. This phase contains physical effects, namely the so-called vacuum polarization which is produced by the external field. Therefore, it is needed to complete the construction of the S-matrix, and here is the place where the causality condition comes in for the first time. With this experience we are then able to construct the S-matrix of full QED by causal perturbation theory in Chap. 3. The important point is that this directly leads to the finite ("renormalized") perturbation series. In fact, no divergent Feynman integral and no ultraviolet cutoff will appear in this book, explaining why the title "Finite QED" was chosen.

It is a common belief that QED with a cutoff or scale parameter should be considered as part of a more fundamental theory where the scale parameter disappears, and that the theory is only mathematically well defined in this bigger framework. We will see that there is no scale parameter in QED in the causal approach if the electrons are massive. If one considers massless fermions, then a scale parameter appears in a natural way, because the central splitting solution (Sect. 3.2) no longer exists. This suggests that it seems indeed necessary to study a bigger theory if one wants to attack the mass problem. But if we take the electron mass as a given finite parameter, QED still has a good chance of being well defined. In fact, the perfect agreement of the perturbative results with experiment cannot be an accident; there must exist well-defined objects (perhaps the adiabatically switched S-matrix $S(g)$ of Sect. 3.1) which are approximated by perturbation theory.

The fact that the causal theory is perturbative has not only a technical but also a deep physical reason. In any realistic quantum field theory one must draw a sharp distinction between the fundamental fields that appear in the elementary interaction and the asymptotic states describing the real incoming and outgoing particles. This is well known today from the theory of strong interactions (quantum chromodynamics, QCD) where the quark fields are the fundamental Fermi fields, while the mesons and nucleons are complicated bound states of them. But even the electron is complicated because it carries the Coulomb field, so it must be regarded as a bound state where (scalar) photons are confined to a Dirac field. Compositeness is the normal case. Only the photon and the neutrini seem to be elementary in the sense that they can simply be generated by fundamental fields. In the causal theory the very hard problem of the asymptotic states is clearly separated from the rest of the theory by the method of adiabatic switching: the interaction is multiplied by a test function $g(x)$ and one performs the adiabatic limit $g \to 1$ at the very end in observable quantities. This means that the confinement is switched off in the asymptotic region in a "gedanken-experiment", so that free fields are coming out, instead of the complicated real physical particles. The switching is then removed in the adiabatic limit. From the study of this limit one can learn something about the structure of the real asymptotic states. It turns out that the limit does not always exist. Only if the right inclusive cross sections are considered does the limit come out finite and unique. In this way the S-matrix itself dictates the structure of the physical particles, as it must be. This highly important fact, which is even more important in non-abelian theories, can already be seen in perturbation theory, as we will discuss at the end of Chap. 3. But it seems to be rather hopeless to jump by some non-perturbative guess directly to the correct description of the asymptotic states.

The inductive construction of the S-matrix enables us to give simple inductive proofs of the various properties of the theory, in particular gauge invariance and unitarity. These themes are described in Chap. 4. The discussion of other electromagnetic couplings in Chap. 5. brings in new features which are important for preparing the extension of the causal method to non-abelean gauge theories. One might regret that this subject is not yet included, but the Epilogue gives a short account of the present status of this field. In the historical introduction the various lines of development in quantum field theory are discussed. From the beginning in the 1920s until today this was a fascinating sequence of successes and failures, where each attempt contained its piece of truth.

The book differs considerably from its first edition: Chapter 3 was completely rewritten and the Chaps. 4 and 5 are new. The bibliographical notes give some hints for further reading.

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Zürich, May 1995 *G. Scharf*

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