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Fundamentals of Convex Analysis

With 66 Figures

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Preface

This book is an abridged version of our two-volume opus *Convex Analysis and Minimization Algorithms* [18], about which we have received very positive feedback from users, readers, lecturers ever since it was published – by Springer-Verlag in 1993. Its pedagogical qualities were particularly appreciated, in the combination with a rather advanced technical material.

Now [18] has a dual but clearly defined nature:

– an introduction to the basic concepts in convex analysis,
– a study of convex minimization problems (with an emphasis on numerical algorithms),

and insists on their mutual interpenetration. It is our feeling that the above basic introduction is much needed in the scientific community. This is the motivation for the present edition, our intention being to create a tool useful to teach convex analysis. We have thus extracted from [18] its "backbone" devoted to convex analysis, namely Chaps III–VI and X. Apart from some local improvements, the present text is mostly a copy of the corresponding chapters. The main difference is that we have deleted material deemed too advanced for an introduction, or too closely attached to numerical algorithms.

Further, we have included exercises, whose degree of difficulty is suggested by 0, 1 or 2 stars *. Finally, the index has been considerably enriched.

Just as in [18], each chapter is presented as a "lesson", in the sense of our old masters, treating of a given subject in its entirety. After an introduction presenting or recalling elementary material, there are five such lessons:

– A Convex sets (corresponding to Chap. III in [18]),
– B Convex functions (Chap. IV in [18]),
– C Sublinearity and support functions (Chap. V),
– D Subdifferentials in the finite-valued case (VI),
– E Conjugacy (X).

Thus, we do not go beyond conjugacy. In particular, subdifferentiability of extended-valued functions is intentionally left aside. This allows a lighter book, easier to master and to go through. The same reason led us to skip duality which, besides, is more related to optimization. Readers interested by these topics can always read the relevant chapters in [18] (namely Chaps XI and XII).

During the French Revolution, the writer of a bill on public instruction complained: "Le défaut ou la disette de bons ouvrages élémentaires a été, jusqu'à présent, un des plus grands obstacles qui s'opposaient au perfectionnement de l'instruction. La raison de cette disette, c'est que jusqu'à présent les savants d'un mérite éminent ont, presque toujours, *préféré la gloire d'élever l'édifice de la science à la peine d'en éclairer l'entrée.*[1]" Our main motivation here is precisely to "light the entrance" of the monument Convex Analysis. This is therefore not a reference book, to be kept on the shelf by experts who already know the building and can find their way through it; it is far more a book for the purpose of learning and teaching. We call above all on the intuition of the reader, and our approach is very gradual. Nevertheless, we keep constantly in mind the suggestion of A. Einstein: "Everything should be made as simple as possible, but not simpler". Indeed, the content is by no means elementary, and will be hard for a reader not possessing a firm mastery of basic mathematical skill.

We could not completely avoid cross-references between the various chapters; but for many of them, the motivation is to suggest an intellectual link between apparently independent concepts, rather than a technical need for previous results. More than a tree, our approach evokes a spiral, made up of loosely interrelated elements.

Many sections are set in smaller characters. They are by no means reserved to advanced material; rather, they are there to help the reader with illustrative examples and side remarks, that help to understand a delicate point, or prepare some material to come in a subsequent chapter. Roughly speaking, sections in smaller characters can be compared to footnotes, used to avoid interrupting the flow of the development; it can be helpful to skip them during a deeper reading, with pencil and paper. They can often be considered as additional informal exercises, useful to keep the reader alert.

The numbering of sections restarts at 1 in each chapter, and chapter numbers are dropped in a reference to an equation or result from within the same chapter.

Toulouse and Grenoble,
March 2001 J.-B. Hiriart-Urruty, C. Lemaréchal

[1] "The lack or scarcity of good, elementary books has been, until now, one of the greatest obstacles in the way of better instruction. The reason for this scarcity is that, until now, scholars of great merit have almost always preferred the glory of constructing the monument of science over the effort of lighting its entrance." D. Guedj: *La Révolution des Savants*, Découvertes, Gallimard Sciences (1988) 130 – 131.

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