Heat and Mass Transfer

Series editors

D. Mewes, Hannover, Germany
F. Mayinger, Garching, Germany

For further volumes:
http://www.springer.com/series/4247

Achintya Kumar Pramanick

The Nature of Motive Force

Springer

Achintya Kumar Pramanick
Department of Mechanical Engineering
National Institute of Technology Durgapur
West Bengal
India

ISSN 1860-4846 ISSN 1860-4854 (electronic)
ISBN 978-3-642-54470-5 ISBN 978-3-642-54471-2 (eBook)
DOI 10.1007/978-3-642-54471-2
Springer Heidelberg New York Dordrecht London

Library of Congress Control Number: 2014932827

© Springer-Verlag Berlin Heidelberg 2014
This work is subject to copyright. All rights are reserved by the Publisher, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, reuse of illustrations, recitation, broadcasting, reproduction on microfilms or in any other physical way, and transmission or information storage and retrieval, electronic adaptation, computer software, or by similar or dissimilar methodology now known or hereafter developed. Exempted from this legal reservation are brief excerpts in connection with reviews or scholarly analysis or material supplied specifically for the purpose of being entered and executed on a computer system, for exclusive use by the purchaser of the work. Duplication of this publication or parts thereof is permitted only under the provisions of the Copyright Law of the Publisher's location, in its current version, and permission for use must always be obtained from Springer. Permissions for use may be obtained through RightsLink at the Copyright Clearance Center. Violations are liable to prosecution under the respective Copyright Law.
The use of general descriptive names, registered names, trademarks, service marks, etc. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant protective laws and regulations and therefore free for general use.
While the advice and information in this book are believed to be true and accurate at the date of publication, neither the authors nor the editors nor the publisher can accept any legal responsibility for any errors or omissions that may be made. The publisher makes no warranty, express or implied, with respect to the material contained herein.

Printed on acid-free paper

Springer is part of Springer Science+Business Media (www.springer.com)

Decisively Dedicated
To
My Parents,
My Guru Srimat Swami Paramananda
and
My Better Half

"Perhaps this monsoon has poured more;
More than a sobbing soul before a mirage.
Bushes are hiding a secret love
Under the bosom of damsel earth.
Didn't you listen the nocturnal flute?

The lusty green grasses in my lawn
Smiling with unusual shyness.
The virgin hibiscus is still giggling
For she is untouched tonight.
Can't you smell the seducing fragrance?

The morning breeze is heavy today.
The temple songs are more melodious.
So I forgo to prepare my lecture notes:
The diary of a man's search.
Why don't you see the chariot is ready?"

Achintya Kumar Pramanick

When I heard the learn'd astronomer,

When the proofs, the figures, were ranged in columns before

me,

When I was shown the charts and diagrams, to add, divide,

and measure them,

When I sitting heard the astronomer where he lectured with

much applause in the lecture-room,

How soon unaccountable I became tired and sick,

Till rising and gliding out I wander'd off by myself,

In the mystical moist night-air, and from time to time,

Look'd up in perfect silence at the stars.

W. Whitman

Foreword

Professor Pramanick's "The Nature of Motive Force" is a delightful walk through the garden of thermodynamics and design in nature. For those who know thermodynamics, this book and its many ideas, quotes, and references are a treat. For those who are curious and eager to know, this is a very attractive invitation.

The law of motive force is the general observation that one or more tradeoffs happen when an effort is made to effect a change. From such tradeoffs emerge the features of organization that persist (shape, dimensions, structure, rhythm). This natural tendency is illustrated with numerous examples from thermal sciences: thermodynamics, heat transfer, and fluid mechanics.

"The Nature of Motive Force" is a treatise on the beauty and permanence of thermodynamics. It puts together several contemporary advances such as the constructal law, the intersection of asymptotes, entropy generation minimization, and convection fundamentals. The book derives its strength from Prof. Pramanick's erudition and strong grasp of mathematics, thermodynamics, languages, and history. I recommend this book very strongly.

Adrian Bejan
J. A. Jones Distinguished Professor
Duke University

Preface

'Tis the good reader that makes the good book; in every book he finds passages which seem confidences or asides hidden from all else and unmistakenly meant for his ear; the profit of books is according to the sensibility of the reader; the profoundest thought or passion sleeps as in a mine, until it is discovered by an equal mind and heart.

R. W. Emerson

I was always very curious about how one gets around to preparing a book and so perhaps as a reader you are as well. Mark Twain takes me into his confidence with the words: "…there ain't nothing more to write about, and I am rotten glad of it, because if I'd'a' knowed what trouble it was to make a book I wouldn't'a' tackled it, and ain't a-going to no more." In contemporary practice portraying a prelude many not have *argumentum ad hominem* much in vogue. But in the entire gamut of my reading experience, I never laid hands on any treatise without going through the very pursuit of the author first. On the same ground, it remains almost a compelling choice for me to insinuate what inspires me. I plead to exempt me from the *egalitarian fallacy* of trying to make all persons alike.

Every true research is but autobiographical and so is the following monograph. At a personal level, trying the best to be very meticulous and carping on almost every aspect that crops up in my way even results in imperfect performance, and thus further suffering a setback of dilemma on decision. Riding on the lacuna of my habit of witnessing ill decision and the stigma of a perfectionist, I was prompted to compose my first scientific writing [1] while I was a second-year undergraduate student in 1991, of a 4-year Mechanical Engineering degree program at the National Institute of Technology Durgapur, India, formerly recognized as Regional Engineering College Durgapur. After many years of latency, in January 2007, I submitted my doctoral thesis [2] haunted by my way of dogma and dilemma and by June 2007 I defended. In 2009, my doctoral thesis was selected only in the group of top five by the Prigogine prize selection committee for the best doctoral thesis in thermodynamics and hence my work could not see the delightful sun of scrutiny by a wide range of readers. Today, I continue to regard that my scientific approach has not been well circulated, especially among physicists. Until in January 2012, when I got a call from Adrian Bejan to publish a book chapter [3], I did not get a pat on my back. By now I got older and somewhat more immuned and case hardened about what other people would think of my preparation and presentation. Granted by Heaven, maybe I can afford to toy myself with the fascinating idea of writing a book. Jonathan Swift has rightly pointed out that, “if a Writer would know how to behave himself with relation to Posterity; let him consider in old Books, what he finds, that he is glad to know; and what Omissions he most laments.”

The essay by Sadi Carnot [4], about a quarter of a century earlier than the terminology adopted by Thomson [5], is a milestone example of how the proponent of a new theory has no choice but to misuse the language of old theory [6]. Thus, without a constant misuse of language there cannot be any discovery, any progress [7]. Tuesdell [8, 9] addressed the celebrated failure of thermodynamics in the nineteenth century, accursed by misunderstanding, irrelevance, and retreat. In the spreaded span of the late twentieth century to the beginning of the twenty-first century, through the constructal theory (fourth law of thermodynamics) [10–12] proposed by Bejan, a consistent brilliant progress has been made in the unified description of nature as well as artificial (engineered) systems. Leib and Yngvason, [13] for the first time in the history of thermodynamics, made it scholarly possible to realize the concept of entropy purely on a macroscopic basis, in contrast with the system theoretic approach of thermodynamics by Haddad et al. [14]. In company with these recent developments, the present treatise is a systematic development and application of a new theory of motive force (power), long due after Carnot [15, 16]. The former faint ideas of the author, which go by the label “heuristic” [17] and “method of synthetic constraint” [18], are formally forged into a generalized formulation recognized as a natural tendency and hence perhaps may be regarded as a law of nature.

The crisis of totalitarian victory is, from the perspective of history, an awkward predicament characterized by intellectual sloth, lack of imagination, and wishful thinking [19]. It is well known that no science develops systematically from one single starting point according to a definite preconceived plan, but its development depends on practical considerations and proceeds more or less simultaneously along different lines, corresponding to the many ways of looking at the problems, and to the times and views of the investigator. Thus, science cannot attain its objective by direct means, but only gradually along numerous and devious paths, and therefore a wide scope is provided for the individuality of the worker [20].

Admittedly, if we do not succeed in solving a mathematical or physical problem, it is often because we have failed to recognize the more general standpoint from which the problem before us appears as a single link in a chain of related problems. This way to find generalized methods is certainly the most practical and the surest one, for he who seeks the method without having a definite problem in mind seeks in vain [21]. For reasons, in this present endeavor, it is preferable to choose some topical problems that are of common interest both to physicists and to the engineering community. This effort is inspired by the seminal call of Bejan through a letter [22] that appeared in the Journal of American Association of Physics Teachers (AAPT).

Nowadays, it has become a fashionable trend [23–25] to publish volumes of empirical material without any thesis or antithesis, such as figures, photographs, computer generated images, and essays on the observation that both natural and engineered systems exhibit a category of symmetry [26, 27]. In contrast, this monograph is a submission against such strategies that may eventually open up the vision of contemporary as well as the next generation of researchers. This, at any rate in my opinion, lack or even absence of figures accompanying the analysis or description, actually stimulates the abstract thinking process, which is eventually the key to the problem solving aspect.

The purview of this current script is to purport a commonality of a diverse view of observations. At the present state of human knowledge and affairs, such a unified exact description of everything on a general footing, both at the macroscopic as well as microscopic [28] levels, will only be poor, vague, and scanty. A theory that is too general is frequently too weak. There is a way out, which I frequently describe to my disciples as an approach from the "periphery to the center." We must not look at the intricacies of the objects and events at the outset; we will keep a habit of looking into simplicities out of complexities and thus concentrate on the outer aspects of the subject during introspection. We will go on adding details in succession until we are undone with a realistic solution to the problem at our disposal. Thus, we will be able to compare apples to oranges. Dwelling on this qualitative aspect, everything appears to be a ramification of a single principle and a unique perspective, which is the object of the present treatise.

This memoir is the faithful disposition of a discourse that I witnessed and withstood with pain and pleasure as a rational as well as an emotional being. Education is the manifestation of perfection already present in man. This study is a passage to that destiny of freedom: from bondage to spiritual faith, from spiritual faith to great courage, from courage to liberty, from liberty to abundance, from abundance to selfishness, from selfishness to complacency, from complacency to apathy, from apathy to dependency, from dependency to back to bondage again. Where and how [29] do I break the chain? The teleological perspective [30, 31] of the present work that tacitly follows in disguise is however not a theme of this treatise. The highest motto underlying the curtain of thorough scientific investigation is but a true aspiration for self-knowledge and self-realization [32], or at least the awakening and sharpening of human faculties already attributed to us. For any conceivable physical principle, there must be a corresponding counterpart of mental (psychological) principle, which in turn is a replica of a metaphysical (spiritual) principle. Our complete realization will actually mean an assimilation of a principle distinctly at these three different levels of human perception. One will then at least be able to rejoice in an added confidence in thought, speech, and action [33]. These exercises were part and parcel of the character of the founders of modern science [34, 35]. An earnest study habit [36] will enable the reader to attain a greater vision to see, which is attributed only at the elevated consciousness [37]. For example, the clairvoyant investigations [38] into the structure of matter carried out by theosophists Besant and Leadbeater was confirmed [39, 40] by the physicist Philips through experimentation and scientific reasoning. As a matter of passing mention, a reader can check the progress on his way to attainment: while in deep thought (meditation), in a single chance you are able to look up a topic from a book without consulting the index first. Accordingly, I have adopted the following principles in my research, with *noblesse oblige*.

Research requires curiosity, diligence, devotion, and aimful thinking [41, 42]. The Latin *mundo corde* describes it better. One can learn so much out of anything, if one can truly start with a blissful ignorance [43]. A perished thought is surely a germinating one. Many a sleepless thoughtful night can give birth to a resourceful dawn. Also, a researcher has first much to do with the overcoming of one's own inferiority complex [44, 45]. As such, one should publish a piece of work when it is even imperfect and incomplete than perfect and complete never. To start with, topics may be chosen with reference to some works of authorities on the subject field. Refinement, generalization, and/or dismissal of their findings could be found as a means of gaining confidence in the research progress.

There are at least two distinct ways in which a subject field can be developed. One is the "horizontal" expansion into the more remote fields intersected by the subject. Another is the "vertical" expansion, that is, a deepening of our present understanding (inception, conception, and the perception) that defines the province. A large number of contemporary workers continue to regard the field of classical thermodynamics as matured and saturated; that is precisely why such old and prevalent topics are picked up. There remains not only a merit in questioning the established point of view, but also the fact that a true research frontier is, quite often, a territory overlooked by the crowd [46]. For such reasons the classical and fundamental research is sought, so that we learn to answer the question "why" and not "how" alone [47]. In every inch of the work, a good balance between the case-specific subjective findings and the general objective reality [48] of a scientific query of general nature [49] is being established.

Regarding the research publication guideline, a piece of advice by Moran was followed [50]: (a) If the work is in the realm of theory, then what truly new insights or relations are achieved, and what is their importance? (b) If the endeavor is in the realm of engineering, then what is the contribution? (c) Does the development provide at least a picture book engineering pointing the way to a significant evolution in some aspect of engineering practice? Also, the present research constantly haunted a physical principle devoid of many computer produced tables and graphs [25]. It was the untiring motivation of the current investigation that the purpose of the computing is insight but not numbers [51].

In my stride I am blissfully aware of the very presence of my masters who preached me to inculcate discipline, method, scholarship, taste, and style up to an adorable personality, for my own survival and succor. For such reasons, it is perhaps not untimely and unprovoked to support the view [52]: "It is recorded that Sancho Panza, when he saw his famous master charge into the windmills, muttered in his beard something about relative motion and Newton's Third law. Sancho was right: the windmills hit the master just as hard as he hit them."

Some peers may rate my presentation as rushed, unwise, and inordinately pretentious and abstract. Perhaps it is. But it is too late now to escape the influence of my masters who taught me that prudence is a rich old maid courted by incapacity. Should they pay heed to the warnings of William Blake?

"When all their Crimes, their Punishments, their Accusation
of Sin,
All Jealous Revenges, Murders, hiding of Cruelty
in Deceit
Appear only at the outward Sphere of Visionary Space and
Time
In the shadows of Possibility, by Mutual Forgiveness for
Evermore
And in the Visions & in the Prophecy, that we may Foresee
& Avoid
The terrors of Creation, Redemption & Judgment."

Durgapur, India — Achintya Kumar Pramanick
Guru Purnima: July 2013

References

1. Pramanick, A.K.: Philosophy of nature. Reflection magazine, pp. 2–5. R.E. College, Durgapur (1990–1991)
2. Pramanick, A.K.: Natural philosophy of thermodynamic optimization. Doctoral thesis (Unpublished), Indian Institute of Technology, Kharagpur (2007)
3. Pramanick, A.K.: Equipartition of Joulean heat in thermoelectric generators. In: Rocha, L.A.O., Lorente, S., Bejan, A. (eds.) Constructal Law and the Unifying Principle of Design. Springer, New York (2013)
4. Klein, M.J.: Carnot's contribution to thermodynamics. Phys. Today **27**, 23–28 (1974)
5. Thomson, W.: Mathematical and Physical Papers-I. Cambridge University Press, London (1882)
6. Bejan, A.: Advanced Engineering Thermodynamics, p. 50. Wiley, New York (1997)
7. Feyerabend, P.: Against Method, p. 27. Verso, London (1978)
8. Truesdell, C.: The Tragicomedy of Classical Thermodynamics. CISM, Udine, Courses and Lectures, No. 70. Springer, New York (1983)
9. Truesdell, C.: Rational Thermodynamics, pp. 1–57. Springer, New York (1984)
10. Bejan, A.: Shape and Structure, from Engineering to Nature. Cambridge University Press, Cambridge (2000)
11. Bejan, A., Zane, J.P.: Design in Nature: How the Constructal Law Governs Evolution in Biology, Physics, Technology, and Social Organization. Anchor Books, New York (2013)
12. Bejan, A.: Advanced Engineering Thermodynamics, p. 807. Wiley, New York (1997)
13. Lieb, E.H., Yngvason, J.: The physics and mathematics of second law of thermodynamics. Phys. Rep. **310**, 1–96 (1999)
14. Haddad, W.M., Chellabonia, V.S., Nersesov, S.G.: Thermodynamics: A Dynamical Systems Approach. Princeton University Press, New Jersey (2005)
15. Mendoza, E. (ed.): Reflections on the Motive Power of Fire. Dover, New York (2005)
16. Kestin, J. (ed.): The Second Law of Thermodynamics-I. Dowden, Hutchinson and Ross, Pennsylvania (1976)
17. Pramanick, A.K., Das, P.K.: Heuristics as an alternative to variational calculus for optimization of a class of thermal insulation systems. Int. J. Heat Mass Transf. **48**, 1851–1857 (2005)
18. Pramanick, A.K., Das, P.K.: Method of synthetic constraint, Fermat's principle and the constructal law in the fundamental principle of conductive heat transport. Int. J. Heat Mass Transf. **50**, 1823–1832 (2007)

19. Bridgman, P.W.: A challenge to physicists. J. Appl. Phys. **13**, 209 (1942)
20. Planck, M.: A Survey of Physical Theory (trans: Jones, R., Williams, D.H.), p. 82. Dover, New York (1993)
21. Hilbert, D.: Mathematical Problems. Arch. Math. Phys. **3**(1), 44–63, 213–237 (1901)
22. Bejan, A.: Engineering advances on finite-time thermodynamics. Am. J. Phys. **62**, 11–12 (1994)
23. Bejan, A.: Advanced Engineering Thermodynamics, p. x. Wiley, New York (1997)
24. D' Le Alembert, J.R.: Nouvelles expériences sur la résistance des fluids. Lambert, Paris (1777) (in French)
25. Truesdell, C.: Six Lectures on Modern Natural Philosophy, pp. 100–101. Springer, New York (1966)
26. Weyl, H.: Symmetry. Princeton University Press, New Jersey (1983)
27. Feynman, R.: The Character of Physical Law, pp. 84–107. MIT Press, Cambridge (1985)
28. Čápek, V., Sheehan, D.P.: Challenges to the Second Law of Thermodynamics: Theory and Experiment. Springer, New York (2005)
29. Dubrovsky, D.: The Problem of the Ideal: The Nature of Mind and Its Relationship to the Brain and Social Medium (trans: Stankevich, V.). Progress, Moscow (1983)
30. Burtt, E.A.: The Metaphysical Foundation of Modern Science. Dover, New York (2003)
31. Esbenshade Jr., D.H.: Relating mystical concepts to those of physics: some concerns. Am. J. Phys. **50**, 224–228 (1982)
32. Gough, A.E.: The Vaisheshika Aphorism of Kanada: With Comments from Upasakara, of Sankara Misra and the Vivriti of Jaya Narayana Tarkapanchana. Motilal Banarsidass, New Delhi (1976)
33. Whorf, B.L.: Language, Thought, and Reality: Selected Writings. In: Carroll, J.B. (ed.). MIT Press, Cambridge (1956)
34. Leibniz, G.W.: Discourse on Metaphysics and the Monadology (trans: Montgomery, G.R.). In: Chandler, A.R. (ed.). Dover, New York (2005)
35. Newton, I.: Newton's Philosophy of Nature: Selections from His Writings. In: Thayer, H.S. (ed.). Dover, New York (2012)
36. Dvivedi, M.N.: The Yoga Sutras of Patanjali. Motilal Banarsidass, New Delhi (2000)
37. Besant, A.: A Study in Consciousness. Theosophical Publishing House, Adyar (1999)
38. Besant, A., Leadbeater, C.W.: Occult Chemistry. Theosophical Publishing House, Adyar (1951)
39. Phillips, S.M.: Anima: Remote Viewing of Subatomic Particles. Theosophical Publishing House, Adyar (1996)
40. Phillips, S.M.: ESP of Quarks and Superstrings. New Age International, New Delhi (2005)
41. Wiener, N.: Invention: The Care and Feeding of Ideas. MIT Press, Cambridge (1993)
42. Munk, M.M.: My early aerodynamic research—thoughts and memories. Ann. Rev. Fluid Mech. **13**, 1–7 (1981)
43. Hopfield, J.J., Feinstein, D.I., Palmar, R.G.: 'Unlearning' has a stabilizing effect in collective memories. Nature **304**, 158–159 (1983)
44. Besant, A.: Thought Power: Its Control and Culture. Quest Books, Illinois (1988)
45. James, W.: Talks to Teachers on Psychology and to Students on Some Life's Ideals. Dover, New York (2001)
46. Bejan, A.: Advanced Engineering Thermodynamics, p. xv. Wiley, New York (1997)
47. Lythcott, J.: "Aristotelian" was given the answer, but what was the question? Am. J. Phys. **53**, 428–432 (1985)
48. Born, M.: Natural Philosophy of Cause and Chance, p. 218. Dover, New York (1964)
49. Thompson, B.: An enquiry concerning the source of heat which is excited by friction. Philos. Trans. R. Soc. Lond. **88**, 80–102 (1798)
50. Moran, M.J.: On second-law analysis and the failed promise of finite-time thermodynamics. Energy **23**, 517–519 (1998)
51. Hamming, R.: Numerical Methods for Scientist and Engineers, p. 3. Dover, New York (1987)
52. Den Hartog, J.P.: Mechanics, p. v. Dover, New York (1961)

Acknowledgments

> *A good many times I have been present at gatherings of people who by the standards of the traditional culture are thought highly educated and who have with considerable gusto been expressing their incredulity at the illiteracy of scientists. Once or twice I have been provoked and have asked the company how many of them could describe the second law of thermodynamics. The response was cold; it was also negative. Yet I was asking which is about the scientific equivalent of: "Have you read a work of Shakespeare's?"*
>
> C. P. Snow

The investigation reported in this memoir spans for about two and half decades in three different continents, viz., India (National Institute of Technology Durgapur, Jadavpur University, and Indian Institute of Technology Kharagpur), the United States of America (Louisiana State University, Baton Rouge), and Europe (Technische Universität Chemnitz, Germany). I connote my gratitude to the authorities of these educational institutions and a few other academias where I professed, as they offered me a great opportunity for research study with their subsequent financial support in terms of fellowships, scholarships, and salaries. I would like to express a token of appreciation to the laboratory incharges and their supporting staff for allowing me to work round the clock without any hindrance or interference. The librarians and coordinating staff of these institutes provided exemplary assistance and cooperation by furnishing me with an exceedingly large volume of documents through interlibrary loan services.

I do not know how to acknowledge my mentors Prof. Sukamal Ghosh from National Institute of Technology (NIT) Durgapur, Prof. Achintya Kumar Mukhopadhyay and Prof. Swarnendu Sen from Jadavpur University (JU), Prof. Srinath V. Ekkad from Louisiana State University (LSU), Prof. Prasanta Kumar Das from Indian Institute of Technology (IIT) Kharagpur, and Prof. Karl Heinz Hoffmann from Technische Universität (TU) Chemnitz. Any word of thankfulness will perhaps be a misnomer. I am much obliged that they agreed to supervise my researches and allowed me to pursue simultaneously a second vocation in new, fundamental, and challenging areas of contemporary interests of mine all along in thermodynamics. It is their meticulous surveillance and close tutelage that enabled me to conclude this philosophical work in my lifetime. I recall how my supervisors persistently motivated even during the difficult moments of my research. I cherish their kind support being my friend, philosopher, and guide in all ways.

IIT Kharagpur imparted a major impact on my research career. To begin with, I was a visiting fellow at Center for Theoretical Studies (CTS) for a short period. During the lean period between the submission of my doctoral thesis in January 2007, and until the acceptance of this proposed monograph on November 2012, my bosom friend Dr. Partha Pratim Bandyopadhyay from IIT Kharagpur continued to invite me for a collaborative work at CTS and thus kept my spirit kindled. The rest of the void in my creativity was partially mitigated by Prof. Gautam Biswas, Director of Central Mechanical Engineering Research Institute (CMERI) Durgapur, India.

Professor Karl Heinz Hoffmann from Institute of Physics, Computational Physics group, TU Chemnitz was a magnanimous host in my research career. I enjoyed inculcating the freedom of ideas in his company spread over almost one and a half years. Working with him was more of a pleasure than a privilege alone. His critical but constructive criticism persuades me to adopt an active research career in physics and particularly in quantum thermodynamics. Miss Angelique Gaida, secretary to Prof. Hoffmann, was very kind and prompt in replenishing the materials required for research, including a very large number of books from time to time.

With the advent of a rapid communication system, a number of international academic personalities provided me with their authored treatises and/or the reprint of articles completely free of cost and finally lent their attention for consultation in a number of occasions. Among a host, in alphabetical order pertinent to my work, I thankfully acknowledge the generosity of Prof. Alexis De Vos, Universiteit Gent, Belgium; Dr. Anatoly Tsirlin, Program Systems of Russian Academy of Sciences, Russia; Prof. André Thess, Technische Universität Ilmenau, Germany; Prof. Bernard Howard Lavenda, Università Camerino, Italy; Prof. Bjarne Andresen, University of Copenhagen, Denmark; Prof. Dick Bedeaux, Norwegian University of Science and Technology, Norway; Prof. Elias Panayiotis Gyftopoulos, Massachusetts Institute of Technology, USA; Prof. Gian Paolo Beretta, Universitá de Brescia, Itally; Prof. Hans Ulrich Fuchs, Zurich University of Applied Sciences at Winterthur, Switzerland; Prof. Ingo Müller, Technische Universität Berlin, Germany; Prof. Jeffery Lewins, University of Cambridge, UK; Prof. Jeffrey M. Gordon, Ben-Gurion University of the Negev, Israel; Prof. Jincan Chen, Xiamen University, China; Prof. Massoud Kaviany, University of Michigan, Ann Arbor, USA; Prof. Michel Feidt, Institut National Polytechnique de Lorraine et Universite Henri Poincare, France; Prof. Peter Salamon, San Diego State University, USA; Prof. Richard Stephen Berry, University of Chicago, USA; Prof. Signe Kjelstrup, Norwegian University of Science and Technology, Norway.

Old wine tastes better. A large number of seasoned friends, former students, and teachers continue to be on my stride. Residing well outside the give-and-take relationship, they flare up my tempo. I still bask in the support and encouragement rendered by Prof. Sumanta Acharya from LSU, Baton Rouge, USA. I could not escape the everlasting influence of Prof. Samir Kumar Saha, from Jadavpur

University, who introduced and finally enticed me into research on the fundamental frontiers of thermodynamics while I was a student of my first master's degree.

I forgot to mention that researchers are also emotional creatures. I am bonded to a large number of hail-fellow-well-mate peers through their cordiality and congeniality for sharing the moments of success and failure with equal ease and comfort and thus aptly creating a social environment for sustenance. My interest in spirituality, palmistry, and above all, openly being praised and criticized as well as reciprocating drew them nearer to my heart. Right from attending movie shows to gossiping in a tea stall or near a vending machine are simply filled with immortal and undefiable memories. Without this impulsive and unobtrusive environment I doubt I would have inducted the breath and the food for thought of my research. Accordingly, I engrave my commitment of faithfulness to a large of number of pals. With a responsible fear of missing to mention any one, I abstain from showcasing my ungratefulness.

On a sentimental note, I reveal and reiterate my tender feelings to my parents—my staunch well-wishers, brothers, sisters, and all other family members for their relentless spontaneous support and sacrifice of personal gratifications towards the realization of my own academic ambitions. The lion's share of my thanksgiving is due to my better half for her patient understanding and endless endurance. It was her pleasant duty to awake me whenever my alarm clock were fade up at times. She crosschecked several drafts of the main body of this work and assisted with the placement of references, figures, and the overall organization. However, for any inadvertent error, I bear the sole responsibility.

While conjuring up the past, I am amused at the awful moment when my spiritual Guru Swami Paramanada Maharaj charged my consciousness as if I got a major bash for the first time in life. I learned to some extent to live in this Vanity Fair otherwise being incorrigibly namby pamby and vulnerable in my approach. His uncalled for compassion (*kripa*) composed me to be totally ransacked and devastated out of my mental constitution (*samaskara*). His preaching of polydimensional geometry and to realize up to seven dimensions in this life itself is a continuous persuasion to submit this monograph. He is writing through my pen with this imperfect instrument like me. By the virtue of studentship alone, I took it for granted the enormous encouragement and buttress provided by Prof. Adrian Bejan, Duke University, USA. I started communicating with him about his works through postal letters beginning in October 1997 and by August 2000 I was supposed to be his doctoral student. Somehow, this ambition could not get matured but I continue to be his pupil. This treatise is an inescapable magnetism of him.

An impetus and harbinger to this writing is a series of events beginning with a review of a book in August 2011 and finishing with a publication of a book chapter of mine in January 2013. In the process I became a fan of Springer. On the anvil of time after several communications, in November 2012, I consented to publish this monograph to my publishing editor Dr. Leontina De Cecco and coordinator Dr. Holger Schäpe. I cannot miss this opportunity to express my profound gratitude to Dr. Leontina and Dr. Schäpe, who so spontaneously, patiently, and cheerfully goaded, prodded, pushed, wheedled, and cajoled me into finishing in reasonable time, and above all, adopted my idea.

Although crafting the section on acknowledgments has the advantage of being ocular, it frequently suffers from the stigma that what is seen is only sensed. I sincerely apologize for any omission that might have slipped in oblivion.

Why art thou silent? Is thy love a plant
Of such weak fibre that the treacherous air
Of absence withers what was once so fair?
Is there no debt to pay, no boon to grant?

W. Wordsworth

Contents

Nomenclature

Letters

a	Acceleration of a particle
a	Arbitrary complex number
a	Constant [Eq. (2.43)]
a_m	Constant
A	Area
A	Cross-sectional area of a fluid column
A	Cross-sectional area of a leg of a thermoelectric module
A	Dimensionless parameter [Eq. (3.35)]
A	Heat exchanging surface area
A	Total heat exchanger surface area
ΔA	Elemental surface area
A_H	Hot end heat exchanger surface area
A_l	Area of the lower strip
A_L	Cold end heat exchanger surface area
A_u	Area of the lower strip
b	Constant [Eq. (2.43)]
b	Dimensionless parameter [Eq. (3.33)]
b	Nonzero complex number
B	Dimensionless parameter [Eq. (3.35)]
Bi	Biot number [Eq. (3.6)]
Br_x	Local Brun number [Eq. (2.45)]
c	Constant heat capacity
c_i	Causal factors, $i = 1, 2, \ldots, n$ [Eqs. (1.5–1.7)]
c_i	Parametric constants, i = 1, 2, 3, …, 22
C	Centroid
C	Constant [Eq. (1.48)]
C	Constant [Eq. (2.43)]
C	Convection term [Eq. (1.17)]
C	Dimensionless parameter [Eq. (4.35)]
C	Finite constant [Eqs. (5.41a, 5.41b)]
C_a	Constant [Eq. (1.1)]

C_A	Finite constant [Eq. (5.41d)]
$\bar{C}_a$	Constant
C_B	Finite constant [Eq. (5.41f)]
C_h	Constant [Eq. (1.14)]
C_i	Internal thermal conductance of the power plant
C_K	Finite constant [Eq. (5.41c)]
C_r	Constant [Eq. (1.4)]
$\bar{C}_r$	Constant
C_{ra}	Constant [Eq. (2.2b)]
C_{rv}	Constant [Eq. (2.1b)]
C_s	Constant [Eq. (1.3)]
$\bar{C}_s$	Constant
C_{sa}	Constant [Eq. (2.2a)]
C_{sv}	Constant [Eq. (2.1a)]
C_T	Constant [Eq. (2.31)]
C_Z	Finite constant [Eq. (5.41e)]
d	Diffusion like flow quantity
D	Diffusion term [Eq. (1.17)]
Ec	Eckert number
f	Function [Eq. (1.32)]
f	Function [Eq. (1.50)]
f	Function [Eq. (1.82)]
f	Function obtained from Blasius solution [Eq. (4.11)]
$\hat{f}$	Force field
f_b	Backward motivation [Eq. (1.2)]
f_f	Forward motivation [Eq. (1.2)]
F	Function of insulation thickness [Eq. (2.15a)]
F	Shorthand for an integrand [Eq. (3.23)]
F	Thrust on elemental fluid area [Eq. (4.53)]
F_a	Applied force [Eq. (1.8)]
F_b	Backward motivation [Eq. (1.1)]
F_e	Effective force [Eq. (1.12)]
F_f	Forward motivation [Eq. (1.1)]
F_H	Fraction of Joulean heat affecting high temperature heat source
F_i	Force of inertia [Eq. (1.10)]
F_k	Kinetic energy [Eq. (1.14)]
F_L	Fraction of Joulean heat affecting low temperature heat sink
F_p	Potential energy [Eq. (1.14)]
F_x	Pressure force in horizontal direction
g	Function [Eq. (1.82)]
g	Gravitational acceleration
g	Known function [Eq. (1.42)]
G_b	Backward motivation [Eq. (1.2)]
G_f	Forward motivation [Eq. (1.2)]

h	Approximate height of the fluid stream before and after hydraulic jump
h	Constraint [Eq. (1.44)]
h	Elemental length of fluid column
h	Local convective heat transfer coefficient
h	Wave length [Eq. (4.49)]
$\bar{h}$	Location of hydrostatic force
h_i	Heat transfer coefficient between fluid stream and cylindrical wall [Eq. (2.32a)]
h_i	Height of a fluid column, $i = 1, 2, 3, \ldots, n$
$\bar{h}_i$	Depth of center of pressure, $i = 1, 2, 3, \ldots, n$
h_L	Heat transfer coefficient at the extreme downstream [Eq. (3.3)]
h_P	Location of hydrostatic force [Eq. (4.35)]
$\bar{h}_P$	Location of hydrostatic pressure with reference to a pole
h_0	Heat transfer coefficient between insulation and ambient [Eq. (2.32a)]
h_1	Height of the fluid stream before hydraulic jump
h_1	Reference height
$h_{1,\min}$	Minimum height of the fluid stream before hydraulic jump [Eq. (4.47)]
h_2	Height of the fluid stream after hydraulic jump
$h_{2,\min}$	Minimum height of the fluid stream after hydraulic jump [Eq. (4.48)]
H	Depth of a gate
H	Height of isothermal fluid column
H	Orthogonal dimension
i	Arbitrary branching level
i	Number of rooms
i	Particular segment
$\hat{i}$	Unit vector
I	Area moment of inertia of a fluid stream
I	Electric current in a branched network
I	Flow current [Eq. (1.52)]
I	Functional [Eq. (1.32)]
I	Integral [Eqs. (1.35, 1.49)]
$\dot{I}$	Functional [Eq. (1.78)]
I_C	Moment of inertia with reference to centroid
I_i	Constant electric current in a branched network, $i = 0, 1, 2$
$\hat{j}$	Unit vector
J	Dimensionless group [Eq. (4.17)]
$\bar{J}$	Dimensionless physical parameter [Eq. (4.16)]
$\bar{J}_{\max}$	Upper ceiling of insulation volume [Eq. (3.32)]
J_q'	Local heat flux
J_x	Electrical current density vector along x-direction
k	Integral constraint [Eq. (1.42)]
k	Local conductivity of insulating material
$\hat{k}$	Unit vector
k_f	Thermal conductivity of fluid

k_i	Thermal conductivity, $i = 1, 2$
k_i	Thermal conductivity of insulating material [Eq. (2.32a)]
k_i'	Modified thermal conductivity, $i = 1, 2$ [Eq. (2.37c)]
k_w	Conductivity of cylindrical wall [Eq. (2.32a)]
k_w	Thermal conductivity of insulating material
K	Constant [Eq. (1.51)]
K	Thermal conductance
K_H	Thermal conductance of high temperature side heat exchanger
K_L	Thermal conductance of low temperature side heat exchanger
L	Lagrangian [Eq. (1.63)]
L	Length of a cylinder
L	Length of a flat plate
L	Length of the leg of a thermoelectric device
L	Thermodynamic distance
L	Wall length
$L_{\max}$	Maximum permissible length of a thermoelectric module [Eq. (5.10a)]
$L_{\min}$	Minimum permissible length of a thermoelectric module [Eq. (5.10b)]
$L_i L_i'$	Length of the ith thermoelectric module
m	Mass of a particle
m	Number of competing mechanisms [Eq. (3.44)]
m	Number of segments
M	Fixed point location
M_i	Parallel faced ith homogeneous medium, $i = 1, 2, 3, \ldots, N$
n	Exponent in heat transfer coefficient relation [Eq. (3.3)]
n	Index of power law [Eqs. (4.31, 6.3, 6.4)]
n	Nondimensional parameter [Eq. (2.14a)]
n	Number of horizontal parts of a fluid column
n	Number of passages at a certain branching level
n	Type of semiconductor material
$\dot{n}$	Population moving per unit time
$\dot{n}''$	Population per unit area and time
n_1	Real part of complex power law index
n_2	Imaginary part of complex power law index
N	Branching levels
N	Net efflux per unit volume [Eq. (1.27)]
N	Number of contiguous parallel faced homogeneous media
N	Number of rooms
Nu_x	Local Nusselt number [Eq. (2.43)]
p	Pressure on elemental fluid element [Eq. (4.54)]
p	Thermodynamic pressure
p	Type of semiconductor material
p_{atm}	Atmospheric pressure
p_i	Pressure of a fluid column, $i = 1, 2, 3, \ldots, n$
p_1	Reference pressure

P	Fixed location on an area
P	Moving point in two-dimensional plane
P	Pole
P	Power output of the engine [Eq. (6.13)]
ΔP	Maximum pressure difference
$\bar{P}$	Dimensionless power [Eq. (6.20)]
P_i	Arbitrary point in a flow field
P_j	Arbitrary point in a flow field
P_r	Real part of dimensionless power [Eq. (6.35)]
Pr	Prandtl number
$\hat{q}$	Heat flux field
q'	Heat transfer rate per unit length [Eqs. (3.5, 3.21)]
q'_{constant}	Heat transfer rate per unit length of constant thickness profile [Eq. (3.39)]
$q'_{\min}$	Minimum heat transfer rate per unit length [Eq. (3.29)]
$q'_{*\min}$	Minimum heat transfer rate per unit length [Eq. (3.10)]
q'_{taper}	Heat transfer rate per unit length of tapered profile [Eq. (3.34)]
q''	Local heat flux [Eq. (3.4)]
Δq	Local heat transfer rate
Δq_i	Heat transfer from ith segment, $i = 1, 2, 3, \ldots, m$
Q	Heat
Q	Volume of a fluid stream
ΔQ	Form of local heat transfer rate [Eq. (2.3)]
$\dot{Q}$	Heat transfer rate duty
$\dot{Q}'$	Unsteady heat transfer rate
$\dot{Q}_H$	Heat transfer rate from the high temperature source
$\dot{Q}_H$	Steady state heat transfer rate supplied by the heat source [Eq. (6.8)]
$\dot{Q}_H^*$	Heat flow rate to the hot end [Eq. (5.12)]
$\dot{Q}_{*H}$	Heat flow rate to the hot end [Eq. (5.23)]
$\dot{Q}_{HC}$	Steady heat transfer rate between work producing compartment and high temperature side [Eq. (6.5)]
$\dot{Q}'_{HC}$	Unsteady heat transfer rate between work producing compartment and high temperature side [Eq. (6.3)]
$\dot{Q}_i$	Steady bypass heat leak through the machine structures [Eq. (6.7)]
$\bar{\dot{Q}}_i$	Dimensionless bypass heat leak
$\dot{Q}_J$	Joulean heat transport rate [Eq. (5.14)]
$\dot{Q}_k$	Conducted heat transport rate [Eq. (5.13)]
$\dot{Q}_L$	Heat transfer rate from the low temperature sink
$\dot{Q}_L$	Steady state heat rejected at the heat sink [Eq. (6.9)]
$\dot{Q}_{LC}$	Steady heat transfer rate between work producing compartment and low temperature side [Eq. (6.6)]
$\dot{Q}'_{LC}$	Unsteady heat transfer rate between work producing compartment and high temperature side [Eq. (6.4)]

$\dot{Q}_{LCE}$	Steady state heat released to the heat sink by endoreversible heat engine
r	Outer radius of a cylindrical wall
r	Volumetric ratio of the high to the low conductive material
R	Electrical resistance
R	Flow resistance [Eq. (1.52)]
R	Rate term [Eq. (1.15)]
R	Resistance to heat flow
R	Universal gas constant
$\hat{R}$	Vectorial quantity
$\Delta\hat{R}$	Small distance
$\bar{R}$	Total conductive and convective resistance [Eq. (3.7)]
R_i	Resistance at ith branching level
R_i	Time independent parallel resistors, $i = 1, 2$
R_l	Electrical resistance of the lower strip
Re_L	Reynolds number at the extreme downstream
R_s	Electrical resistance of any strip element
R_u	Electrical resistance of the upper strip
Re_x	Local Reynolds number
s	Arc length of the path of light
s	Slenderness ratio
s_{opt}	Optimum slenderness ratio
S	Entropy
S	Scalar quantity
S	Shape factor
S	Source term [Eq. (1.15)]
ΔS	Change in scalar quantity [Eq. (1.21)]
S_{cup}	Entropy of the cup
$\dot{S}_{\mathrm{gen}}$	Entropy generation rate
$\bar{\dot{S}}_{\mathrm{gen}}$	Uniform entropy generation rate [Eq. (3.43)]
S_i	Scalar quantity
S_j	Scalar quantity
S_{room}	Entropy of the room
S_s	Shape factor of a rectangular strip
S_{universe}	Entropy of the universe
S_φ	Source term [Eq. (1.31)]
t	Local insulation thickness
t	Passage time of light [Eq. (1.45)]
t	Time
$\bar{t}$	Length-based averaged wall thickness [Eq. (2.16)]
$\bar{t}$	Length-based averaged wall thickness [Eq. (3.2a)]
t_{1l}	Optimal insulation thickness distribution [Eq. (2.21)]
t_{2l}	Optimal insulation thickness distribution [Eq. (2.22)]
t_{3l}	Optimal insulation thickness distribution [Eq. (2.23)]

t_{opt}	Optimal insulation thickness [Eq. (2.35)]
t_{opt}	Optimal insulation thickness [Eq. (3.28)]
t_{taper}	Tapered insulation profile [Eq. (3.33)]
t_w	Thickness of the wall [Eq. (2.32a)]
t_*	Optimal insulation thickness distribution [Eq. (3.9)]
T	Average absolute temperature of a thermoelectric module
T	Temperature distribution function [Eq. (5.1)]
T	Thermodynamic temperature
T	Wall temperature variation
T_0	Ambient temperature
T_0	Temperature at the bottom surface of the plate
T_0	Wall temperature at $x = 0$
T_1	Wall temperature variation [Eq. (2.40)]
T_∞	Free stream temperature
ΔT	Applied temperature gap across a thermoelectric module
ΔT	Constant thermal potential difference
ΔT	Local temperature gradient
ΔT	Maximum temperature difference
ΔT_r	Relative temperature drop term [Eq. (2.44)]
T_f	Local fluid stream temperature
T_h	Real temperature component of the high temperature side [Eq. (6.1)]
T_H	Heat source temperature
T_H	Highest temperature of rooms
T_H	High temperature side temperature
T_{HC}	High temperature level at which the device actually receives the heat
T_{HC}	Transient temperature of the working fluid at the hot end [Eq. (6.1)]
T_{HO}	Time-averaged temperature of the working fluid at the hot end
T_i	Temperature, $i = 1, 2$
T_i	Temperature of the ith room, $i = 1, 2, 3, \ldots, N$
T_l	Real temperature component of the low temperature side [Eq. (6.2)]
T_L	Heat sink temperature
T_L	Low temperature side temperature
T_L	Lowest temperature of rooms
T_L	Wall temperature at $x = L$
T_{LC}	Low temperature level at which the device actually rejects the heat
T_{LC}	Transient temperature of the working fluid at the cold side [Eq. (6.2)]
T_{LO}	Time-averaged temperature of the working fluid at the cold end
T_w	Interfacial wall temperature [Eqs. (3.14a, 3.14b)]
u	Velocity component along the flat plate
u_i	Velocity of light in ith medium, $i = 1, 2, 3, \ldots, N$
U	Overall heat transfer coefficient
U	Total internal energy of the system
UA	Overall thermal conductance [Eq. (6.16)]
U_H	Overall heat transfer coefficient of high temperature side heat exchanger

U_L	Overall heat transfer coefficient of low temperature side heat exchanger
U_∞	Free stream velocity
v	Control volume size
v	Fixed upper volume of the strip
v	Specific volume of the fluid
v	Velocity component normal to the flat plate
v	Velocity of a particle
$\hat{v}$	Velocity vector
Δv	Form of elemental insulation volume [Eq. (2.3)]
v_l	Volume of the lower strip
v_u	Volume of the upper strip
v_1	Reference specific volume
V	Approximate velocity of fluid stream before and after hydraulic jump
V	Potential difference [Eq. (1.52)]
V	Voltage in a branched network
V	Volume of a fluid element
V	Volume of an insulating material
V_1	Velocity before hydraulic jump
V_2	Velocity after hydraulic jump
$\bar{V}$	Nondimensional insulation volume [Eq. (2.24b)]
$\hat{V}$	Vector flux
ΔV	Elemental insulation volume
ΔV	Voltage drop
V_i	Voltage in a branched network, $i = 1, 2$
ΔV_l	Potential drop of the lower strip
ΔV_u	Potential drop of the upper strip
V_x	Component of a vector flux
V_y	Component of a vector flux
V_z	Component of a vector flux
w	Width of the control volume
w	Width of the finite fluid column
W	Wall width
$\dot{W}$	Work output rate
x	Heat exchanger allocation ratio
x	Longitudinal coordinate direction
x	Orthogonal direction
x	Side of a fluid element of parallelogram shape
x	Variable
Δx	Linear dimension
x_i	Abscissa, $i = 0, 1, 2$
x_i	Constant, $i = 0, 1, 2$
x_i	Roots of a quartic equation, $i = 1, 2, 3, 4$ [Eq. (6.41)]
x_{opt}	Optimal heat exchanger allocation ratio
X	Longitudinal dimension of a rectangular block

X Column vector of extensive variables
X_q Conjugate driving force
y Orthogonal direction
y Side of a fluid element of parallelogram shape
y Transformed heat exchanger allocation ratio [Eq. (6.46)]
y Variable
y Vertical coordinate direction
Δy Linear dimension
y_i Arbitrary function in x_i for $i = 0, 1, 2$
y_i Constant, $i = 0, 1, 2$
y_i Ordinate, $i = 0, 1, 2$
Y Comparison function in x [Eq. (1.33)]
Y Lateral dimension of a rectangular block
z Figure of merit of a thermoelectric module [Eq. (5.39a)]
z Orthogonal direction
z Parameter
z Transformed heat exchanger allocation ratio [Eq. (6.50)]
$\bar{z}$ Transformed heat exchanger allocation ratio [Eq. (6.58)]
Δz Linear dimension
z_i Roots of a cubic equation, $i = 1, 2, 3$ [Eq. (6.50)]

Greek Symbols

α Dummy variable [Eq. (3.18)]
α Inclination of a fluid element with the horizontal
α Seebeck coefficient of the material
β Dummy variable [Eq. (3.18)]
$\bar{\chi}$ Total magnitude of all dissipative forces [Eq. (3.44)]
χ_i Arbitrary design variable, $i = 1, 2, 3, \ldots, m$ [Eq. (3.44)]
δ Dimensionless parameter [Eq. (2.26b)]
δ Thickness of heat exchanging medium
δ_T Thermal boundary layer thickness
Δ Dimensionless parameter [Eq. (2.27c)]
Δ Infinitesimal difference
ε Correction factor [Eq. (2.41)]
ε Effectiveness of heat exchanging equipment
ε Parametric constant [Eq. (1.33)]
ε_H Effectiveness of high temperature side heat exchanger
ε_L Effectiveness of low temperature side heat exchanger
ϕ Degree of irreversibility [Eq. (6.10)]
ϕ Intensive property
ϕ_i Angle of the light ray with the normal at ith medium, $i = 1, 2, 3, \ldots, N$ [Eqs. (1.47, 1.48)]
Φ Aggregate integral [Eq. (3.23)]

Φ	Extensive property
γ	Cost of unit conductance
γ	Specific weight of a fluid column
γ_H	Cost of unit conductance of high temperature side heat exchanger
γ_L	Cost of unit conductance of low temperature side heat exchanger
Γ_ϕ	Coefficient of diffusion
η	Arbitrary function in x
η	Similarity variable [Eq. (3.12)]
η	Thermal efficiency of the engine [Eq. (6.15)]
η_r	Real part of the engine efficiency [Eq. (6.72)]
κ	Thermal conductivity of the material
κ_e	Electrical conductivity
κ_l	Lattice thermal conductivity
κ_0	Constant thermal conductivity of a thermoelectric element
λ	Dimensionless parameter [Eqs. (5.6b, 5.15)]
λ	Lagrange multiplier
λ	Numerical and dimensional factor [Eqs. (2.1a, 2.1b)]
$\hat{\lambda}$	Accommodating factor [Eq. (1.1a)]
λ_i	Constant, $i = 1, 2$ [Eqs. (1.57, 1.58)]
λ_i	Numerical and dimensional factor for ith segment, $i = 1, 2, 3, \ldots, m$ [Eqs. (2.1a, 2.1b)]
Λ	Dimensionless parameter [Eq. (5.6a)]
Λ_1	Parametric group [Eq. (2.19)]
Λ_2	Parametric group [Eq. (2.27a)]
Λ_3	Parametric group [Eq. (2.29a)]
Λ_4	Parametric group [Eq. (3.30a)]
μ	Numerical and dimensional factor [Eqs. (2.2a, 2.2b)]
μ_i	Numerical and dimensional factor for ith segment, $i = 1, 2, 3, \ldots, m$ [Eqs. (2.2a, 2.2b)]
ν	Kinematic viscosity
θ	Angle of applied force
θ	Dimensionless temperature [Eq. (5.4a)]
θ	Included angle between two nonparallel sides of a fluid element
θ	Nondimensionalized fluid temperature [Eq. (3.12)]
θ_1	Angle of incidence [Eqs. (2.37b, 2.37c)]
θ_2	Angle of refraction [Eqs. (2.37b, 2.37c)]
θ_*	Dimensionless temperature distribution without Thomson heat [Eq. (5.19)]
θ^*	Dimensionless temperature distribution with Thomson heat consideration [Eq. (5.8)]
ρ	Density of fluid element
ρ	Electrical resistivity of the material
ρ_e	Effective resistivity of any strip element
ρ_i	Density of a fluid column, $i = 1, 2, 3, \ldots, n$
ρ_l	Electrical resistivity of the lower strip element

ρ_u	Electrical resistivity of the upper strip element
ρ_0	Constant electrical resistivity of a thermoelectric element
σ	Electrical conductivity or reciprocal of electrical resistivity of the material
$\dot{\sigma}$	Local entropy production rate
$\dot{\sigma}_{\mathrm{EoEP}}$	Local entropy production rate with equipartition of entropy production
$\dot{\sigma}_{\mathrm{EoF}}$	Local entropy production rate with equipartition of forces
$\dot{\sigma}_{\mathrm{opt}}$	Optimum local entropy production rate
τ	Ratio of high temperature to the low temperature
τ	Temperature ratio spanned by heat source and sink [Eq. (6.18)]
τ	Thomson coefficient of the material
τ_h	Intermediate temperature ratio [Eq. (6.18)]
τ_o	Steady state temperature ratio spanned by working fluid [Eq. (6.18)]
τ_{opt}	Optimized temperature range of the working fluid
ω	Oscillating periodic frequency of thermal wave [Eqs. (6.1, 6.2)]
ξ	Dimensionless leg length of the thermoelectric device [Eq. (5.4b)]
ξ	Dimensionless length of the flat plate [Eq. (3.2b)]
ξ_*	Location of maximum temperature without Thomson heat [Eqs. (5.21, 5.22)]
ξ^*	Location of maximum temperature with Thomson heat [Eqs. (5.9, 5.11a, 5.11b)]
ψ	Numerical and dimensional factor [Eq. (2.3)]
ζ	Dimensionless heat transfer rate [Eq. (5.20)]

Subscripts

C	Centroid
constant	Uniform wall thickness distribution
cup	Pertaining to the cup
e	Quantities of electrical origin
EoEP	Equipartition of entropy production
EoF	Equipartition of forces
gen	Rate of entropy generation
H	Pertaining to the high temperature side
HC	Transient quantities at the hot end
HO	Time-averaged quantities at the hot end
i	Index
i	Number of competing dissipating mechanisms
i	Number of segments considered in a continuous fluid stream
J	Quantities related to component of Joulean heat
l	Lower strip
l	Quantities of lattice thermal origin
L	Quantities related to low temperature sink
LO	Time-averaged quantities at the cold side

m	Dummy variable [Eq. (3.44)]
max	Maximum
min	Minimum
min	Minimum with conjugate formulation [Eq. (3.29)]
$*$min	Minimum with nonconjugate formulation [Eq. (3.10)]
n	Integer number of partition considered in a finite length of fluid element
n	Parameter determining wall temperature curvature
n	n-type material
opt	Optimum
p	p-type material
P	Reference pole
q	Heat
r	Real part of a complex quantity
room	Pertaining to the room
s	Arbitrary strip
t	Time-averaged quantities
$\bar{t}$	Averaged thickness based quantities
taper	Tapered wall thickness distribution [Eq. (3.33)]
u	Upper strip
universe	Pertaining to the universe
x	Along x-coordinate direction
x	Flow direction
1	Refers to wall
2	Pertains to insulation
*	Optimum wall thickness distribution [Eq. (3.9)]
*	Quantities pertaining to without Thomson heat consideration
δ_T	Quantities based on thermal boundary layer thickness

Superscripts

atm	Atmospheric pressure
–	Averaged dimensionless quantity
–	Location of hydrostatic force
–	Transformed quantities
*	Quantities pertaining to Thomson heat consideration
0	Constancy of physical parameter

Symbols

$\langle\rangle$	Averaged quantity
Δ	Change in value

Abbreviations

EoEP	Equipartition of entropy production
EoF	Equipartition of forces
EoTD	Equipartition of temperature difference
EGM	Entropy generation minimization
ETD	Equal thermodynamic distance
FTT	Finite-time thermodynamics
PM	Power maximum