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MARKOV CHAINS

WITH STATIONARY TRANSITION PROBABILITIES

BY

KAI LAI CHUNG

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TO MY PARENTS

Preface

The theory of Markov chains, although a special case of Markov processes, is here developed for its own sake and presented on its own merits. In general, the hypothesis of a denumerable state space, which is the defining hypothesis of what we call a "chain" here, generates more clear-cut questions and demands more precise and definitive answers. For example, the principal limit theorem (§§ I.6, II.10), still the object of research for general Markov processes, is here in its neat final form; and the strong Markov property (§ II.9) is here always applicable. While probability theory has advanced far enough that a degree of sophistication is needed even in the limited context of this book, it is still possible here to keep the proportion of definitions to theorems relatively low.

From the standpoint of the general theory of stochastic processes, a continuous parameter Markov chain appears to be the first essentially discontinuous process that has been studied in some detail. It is common that the sample functions of such a chain have discontinuities worse than jumps, and these baser discontinuities play a central role in the theory, of which the mystery remains to be completely unraveled. In this connection the basic concepts of separability and measurability, which are usually applied only at an early stage of the discussion to establish a certain smoothness of the sample functions, are here applied constantly as indispensable tools. Hence it is hoped that this book may also serve as an illustration of the modern rigorous approach to stochastic processes toward which there is still so much misgiving.

The two parts of the book, dealing respectively with a discrete and a continuous parameter, are almost independent. It was my original intention to write only the second part, preceded by whatever necessary material from the first. As it turned out, I have omitted details of the continuous parameter analogues when they are obvious enough, in order to concentrate in Part II on those topics which have no counterparts in the discrete parameter case, such as the local properties of sample functions and of transition probability functions. It is these topics that make the continuous parameter case a relatively original and still challenging theory.

Markov process is named after A. A. Markov who introduced the concept in 1907 with a discrete parameter and finite number of states.

The denumerable case was launched by KOLMOGOROV in 1936, followed closely by DOEBLIN whose contributions pervade all parts of the Markov theory. Fundamental work on continuous parameter chains was done by DOOB in 1942 and 1945; and in 1951 PAUL LÉVY, with his unique intuition, drew a comprehensive picture of the field. The present work has grown out of efforts to consolidate and continue the pioneering work of these mathematicians. It is natural that I have based the exposition on my own papers, with major revisions and additions; in particular, the first few sections form an expansion of my lecture notes (mimeographed, Columbia University 1951) which have had some circulation. Quite a few new results, by myself and by colleagues subject to my propaganda, have been as it were made to order for this presentation. Historical comments and credit acknowledgments are to be found in the Notes at the end of the sections. But as a rule I do not try to assign priority to fairly obvious results; to do so would be to insult the intelligence of the reader as well as that of the authors involved.

This book presupposes no knowledge of Markov chains but it does assume the elements of general probability theory as given in a modern introductory course. Part I is on about the same mathematical level as FELLER'S *Introduction to probability theory and its applications, vol. 1.* For Part II the reader should know the elementary theory of real functions such as the oft-quoted theorems of DINI, FATOU, FUBINI and LEBESGUE. He should also be ready to consult, if not already familiar with, certain basic measure-theoretic propositions in DOOB'S *Stochastic processes.* An attempt is made to isolate and expose [*sic*]the latter material, rather than to assure the reader that it is useless luxury. The mature reader can read Part II with only occasional references to Part I.

Markov chains have been used a good deal in applied probability and statistics. In these applications one is generally looking for something considerably more specific or rather more general. In the former category belong finite chains, birth-and-death processes, etc.; in the latter belong various models involving a continuous state space subject to some discretization such as queueing problems. It should be clear that such examples cannot be adequately treated here. In general, the practical man in search of ready-made solutions to his own problems will discover in this book, as elsewhere, that mathematicians are more inclined to build fire stations than to put out fires. A more regrettable omission, from my point of view, is that of a discussion of semigroup or resolvent theory which is pertinent to the last few sections of the book. Let us leave it to another treatise by more competent hands.

A book must be ended, but not without a few words about what lies beyond it. First, sporadic remarks on open problems are given in the Notes. Even for a discrete parameter and in the classical vein, a semblance of fullness exists only in the positive-recurrent case. Much less is known in the null-recurrent case, and a serious study of nonrecurrent phenomena has just begun recently. The last is intimately related to an analysis of the discontinuities of continuous parameter sample functions already mentioned. In the terminology of this book, the question can be put as follows: how do the sample curves manage to go to infinity and to come back from there? A satisfactory answer will include a real grasp on the behavior of instantaneous states, but the question is equally exigent even if we confine ourselves to stable states (as in §§ II.17 to 20). This area of investigations has been called the theory of "boundaries" in analogy with classical analysis, but it is perhaps more succinctly described as an intrinsic theory of compactification of the denumerable state space of the Markov chain. There are a number of allusions to this theme scattered throughout the book, but I have refrained from telling an unfinished story. The solicitous voice of a friend has been heard saying that such a new theory would supersede the part of the present treatment touching on the boundary. Presumably and gladly so. Indeed, to use a Chinese expression, why should the azure not be superior to the blue?

Among friends who have read large portions of the manuscript and suggested valuable improvements are J. L. Doob, Henry P. McKean Jr. and G. E. H. Reuter. My own work in the field, much of it appearing here and some of it for the first time, has been supported in part by the Office of Scientific Research of the United States Air Force. To these, and quite a few others who rendered help of one kind or other, I extend my hearty thanks.

January, 1960. K. L. C.

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