

# Lecture Notes in Biomathematics

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Y. Cohen (Ed.)

## Applications of Control Theory in Ecology

Proceedings of the Symposium on Optimal Control Theory  
held at the State University of New York, Syracuse,  
New York, August 10–16, 1986

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## PREFACE

Control theory can be roughly classified as deterministic or stochastic. Each of these can further be subdivided into game theory and optimal control theory. The central problem of control theory is the so called constrained maximization (which--with slight modifications--is equivalent to minimization). One can then say, heuristically, that the major problem of control theory is to find the maximum of some performance criterion (or criteria), given a set of constraints. The starting point is, of course, a mathematical representation of the performance criterion (or criteria)--sometimes called the objective functional--along with the constraints. When the objective functional is single valued (i.e., when there is only one objective to be maximized), then one is dealing with optimal control theory. When more than one objective is involved, and the objectives are generally incompatible, then one is dealing with game theory.

The first paper deals with stochastic optimal control, using the dynamic programming approach. The next two papers deal with deterministic optimal control, and the final two deal with applications of game theory to ecological problems.

In his contribution, Dr. Marc Mangel applies the dynamic programming approach, as modified by his recent work--with Dr. Colin Clark, from the University of British Columbia (Mangel and Clark 1987)\*--to modelling the "behavioral decisions" of insects. The objective functional is a measure of fitness. Readers interested in detailed development of the subject matter may consult Mangel (1985). My contributions deal with two applications of optimal control theory. The first is an application of classical optimal control theory to the reproductive strategy of plants. The second is the so called impulse control. It is applicable to situations where the controlled state (e.g., amount of food consumed) changes impulsively. In both cases the value functional is some measure of fitness, which is to be maximized. Dr. Robert McKelvey's contribution takes a hard look at the "tragedy of the commons" from a game theory perspective, using, specifically, the differential games approach. Finally, the contribution by Drs. Joel Brown and Thomas Vincent applies game theory ideas to the problem of evolutionary stable strategies (ESS). Their theory is more fully developed in the literature cited in their manuscript.

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\*See references in relevant manuscripts.

There is a growing interest among ecologists in the methods of control theory. This interest is matched by that of a small group of applied mathematicians who find ecological problems interesting and challenging. I hope that this volume will further stimulate mutual interest.

Y. Cohen  
St. Paul, Minnesota  
February 1987

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