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Convexity and Optimization in Finite Dimensions I

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Foreword

Dantzig's development of linear programming into one of the most applicable optimization techniques has spread interest in the algebra of linear inequalities, the geometry of polyhedra, the topology of convex sets, and the analysis of convex functions. It is the goal of this volume to provide a synopsis of these topics, and thereby the theoretical background for the arithmetic of convex optimization to be treated in a subsequent volume.

The exposition of each chapter is essentially independent, and attempts to reflect a specific style of mathematical reasoning.

The emphasis lies on linear and convex duality theory, as initiated by Gale, Kuhn and Tucker, Fenchel, and v. Neumann, because it represents the theoretical development whose impact on modern optimization techniques has been the most pronounced. Chapters 5 and 6 are devoted to two characteristic aspects of duality theory: conjugate functions or polarity on the one hand, and saddle points on the other. The Farkas lemma on linear inequalities and its generalizations, Motzkin's description of polyhedra, Minkowski's supporting plane theorem are indispensable elementary tools which are contained in chapters 1, 2 and 3, respectively. The treatment of extremal properties of polyhedra as well as of general convex sets is based on the far reaching work of Klee. Chapter 2 terminates with a description of Gale diagrams, a recently developed successful technique for exploring polyhedral structures.

The first two chapters require only an elementary knowledge of linear algebra and analytic geometry. Some familiarity with basic topological concepts and the theory of real functions, however, will be needed by readers of the remaining chapters.

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