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LINEAR PARTIAL DIFFERENTIAL OPERATORS

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WITH 1 FIGURE

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Preface

The aim of this book is to give a systematic study of questions concerning existence, uniqueness and regularity of solutions of linear partial differential equations and boundary problems. Let us note explicitly that this program does not contain such topics as eigenfunction expansions, although we do give the main facts concerning differential operators which are required for their study. The restriction to linear equations also means that the trouble of achieving minimal assumptions concerning the smoothness of the coefficients of the differential equations studied would not be worth while; we usually assume that they are infinitely differentiable.

Functional analysis and distribution theory form the framework for the theory developed here. However, only classical results of functional analysis are used. The terminology employed is that of BOURBAKI. To make the exposition self-contained we present in Chapter I the elements of distribution theory that are required. With the possible exception of section 1.8, this introductory chapter should be bypassed by a reader who is already familiar with distribution theory.

No attempt has been made to compile a complete bibliography. Most references given are only intended to indicate recent sources for the material presented or closely related topics. In order to show the connection with the classical theory a few references to older literature have also been given. For a much more extensive bibliography of some of the topics studied here we refer to J. L. LIONS, Equations différentielles opérationelles, which has recently appeared in this series.

I am greatly indebted to Professors B. MALGRANGE and P. COHEN who have permitted the inclusion of unpublished results of theirs in sections 5.8 and 8.9 respectively, and to Professor HENRY HELSON who made a very careful revision of the English text. A major part of the work was done at Stanford University, The University of California, The Institute for Advanced Study and The University of Wisconsin while the author was on leave from The University of Stockholm. I wish to express here my gratitude to all these institutions.

Stockholm, March 1963 LARS HÖRMANDER

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