Die Grundlehren der mathematischen Wissenschaften

in Einzeldarstellungen
mit besonderer Berücksichtigung
der Anwendungsgebiete

Band 129

The Mathematical Apparatus for Quantum-Theories

Based on the Theory of Boolean Lattices

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Springer-Verlag New York Inc. 1966

ISBN-13: 978-3-642-46032-6 e-ISBN-13: 978-3-642-46030-2
DOI: 10.1007/978-3-642-46030-2

Softcover reprint of the hardcover 1st edition 1966
Library of Congress Catalog Card Number 66-27977

Title No. 5112

Dedicated to my wife

Dr. Stanisława Nikodým

Preface

The purpose of this book is to give the theoretical physicist a geometrical, visual and precise mathematical apparatus which would be better adapted to some of their arguments, than the existing and generally applied methods. The theories, presented in this book, are based on the theory of Boolean lattices, whose elements are closed subspaces in the separable and complete Hilbert-Hermite-space.

The first paper, in which the outlines of the said mathematical apparatus is sketched, is that of the author: "Un nouvel appareil mathématique pour la théorie des quanta."[1]

The theory exhibited in this paper has been simplified, generalized and applied to several items of the theory of maximal normal operators in Hilbert-space, especially to the theory of multiplicity of the continuous spectrum and to permutable normal operators, based on a special canonical representation of normal operators and on a general system of coordinates in Hilbert-space, which is well adapted not only to the case of discontinuous spectrum, but also to the continuous one.

The normal operators, which can be roughly characterized as operators with orthogonal eigen-vectors and complex eigen-values, constitute a generalization of hermitean selfadjoint and of unitary operators.

The importance of the methods, sketched in the mentioned paper, has been emphasized in the review in the "Zentralblatt für Mathematik", by the physicist G. Ludwig[2] and later applied by him in his book "Die Grundlagen der Quantenmechanik"[3]. The mentioned theory has

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[1] Annales de l'Institut Henri Poincaré, tome XI, fasc. II, pages 49—112. The paper constitutes the content of four lectures by the author: February 4, 6, 11 and 13, (1947) at the Institut Henri Poincaré in Paris.

[2] Bd. 37, 1951, p. 278/279.

[3] Berlin/Göttingen/Heidelberg: Springer-Verlag. Die Grundlehren der Mathematischen Wissenschaften 52. (1954), XII + 460 pp. (see the footnote p. 75).

later been simplified, generalized and applied in several papers by the author. The present book can be considered as a systematic synthesis of them all, with suitable preparations, additions and precise proofs.

It contains many new notions, as the notion of "trace", which are defined, studied, and applied.

The author hopes that this book will be useful not only to physicists but also to mathematicians.

Concerning the Boolean-lattice-approach, the following remarks are in order: J. v. NEUMANN has found interesting relations between the logic of propositions and some behaviour of projectors in the Hilbert-space. Now, if we introduce with M. H. STONE suitable and simple operations on closed subspaces of the Hilbert-space, we can perceive that just the Boolean lattices made out of closed subspaces constitute the suitable, useful translation of the relations mentioned, found by v. NEUMANN, and that the Boolean lattices should be chosen as a convenient background for further developments.

An other source can be found in the modern theory of set-function and of general, abstract integration and measure, created by DE LA VALLÉE-POUSSIN, VITALI, HAHN, RADON and especially by M. FRÉCHET who has generalized the LEBESGUEan theory to abstract sets and general denumerably additive, non-negative and bounded measure.

The above few sources have made it possible to construct a geometrical theory of selfadjoint operators and to extend it to normal operators.

We mention that the original approach to the mathematical part of the theory of quanta, based on matrices, is not adequate, as has been shown by J. v. NEUMANN in his paper: [J. reine angew. Math. 161 (1913)]. The matrices have been replaced by operators in Hilbert-space (F. RIESZ, J. v. NEUMANN, M. H. STONE).

Since we do not require that the reader be familiar with the modern abstract theories, we shall start with a sketch of the theory of Boolean lattices.

The reader is supposed to be familiar with

1) basic properties of the structure of Hilbert-space, with basic properties of hermitean selfadjoint operators, the Hilbert spectral-theorem included,

2) with the theory of Lebesgue's measure and integration,

3) with basic notions of abstract topology and

4) with the notion of an ideal in a commutative ring. The reader is also supposed to know the necessity of discrimination between notions having different logical type e.g. between a set and a set of sets.

We apply the usual notations with the following novelty: we shall sometimes use a dot over a letter, say x, to emphasize that x is a variable, e.g. $f(\dot{x})$ means a function of the variable x, and $f(a)$ means the value of the function at the point a, the symbol $A_{\dot{n}}$ will mean the sequence $\{A_1, A_2, \ldots, A_n, \ldots\}$ and $\dot{n}$ the sequence $1, 2, 3, \ldots, n, \ldots$ of natural numbers.

The book contains 29 chapters, which are only partly depending on one another. They are labelled with letters:

A, A I, B, B I, C, C I, D, D I, E, F, G, H,
J, J I, K, L, M, N, P, P I, Q, Q I, R, R I,
S, T, U, W, W I.

We are including the list of contents of these chapters.

We give the list of references labelled with a fat parenthesis **( )**. The list contains not only the papers which are directly applied in the text, but also all those papers, which have influenced the author with some useful ideas.

Though the "Apparatus" is destinated for physicists, it does not contain direct applications to mathematical problems of physics. The author intends to deal with them in subsequent papers or in another book.

I am owing my thanks to Prof. Dr. HELMUT HASSE and to Prof. Dr. B. L. VAN DER WAERDEN for their kind recommendation of my work to the Springer-Verlag.

I wish to thank the U.S.A.-Atomic Energy Commission and the U.S.A.-Office of Ordnance Research for their financial help in my research related to the book, and especially I am owing my thanks to the U. S. A. National Science Foundation for support through several years. I am owing special thanks to that institution whose grants have made possible the final composition of the book.

In addition to that I express my thanks to the French "Fondation Nationale des Recherches Scientifiques," whose financial aid has made possible my research on the "Apparatus" in 1946—1948, and especially to Professor ARNAUD DENJOY who kindly arranged that financial aid.

But my most hearty thanks I am owing to my wife Dr. STANISŁAWA NIKODÝM, (also a mathematician) whose help in composing the book, proof reading and typing was very great. Without her efficient help it would have been impossible for me to compose the present book.

Finally, I would like to thank the Springer-Verlag and the printers for a beautiful and very clear setting of a quite difficult text.

Utica, N.Y. USA,
July 10. 1966

Dr. OTTON MARTIN NIKODYM

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