

# Partial Differential Equations: Theory, Control and Approximation



Jacques-Louis Lions in 1998

Philippe G. Ciarlet • Tatsien Li • Yvon Maday  
Editors

# Partial Differential Equations: Theory, Control and Approximation

In Honor of the Scientific Heritage  
of Jacques-Louis Lions

 Springer

*Editors*

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# Preface

Jacques-Louis Lions (1928–2001) was an exceptional mathematician, whose lasting influence is still deeply felt all over the world.

He was a universally recognized and admired expert in partial differential equations, to the study of which he has made outstanding contributions regarding not only the theoretical aspects such as existence and uniqueness of partial differential equations, regularity of the solutions, homogenization, and control, but also their numerical analysis and applications to fluid and solid mechanics, oceanography, climatology, etc.

Together with Enrico Magenes, he first produced an exhaustive analysis of linear boundary value problems posed in Sobolev spaces, which includes, in particular, a remarkably elegant proof of Korn's inequality. He then developed with Guido Stampacchia the theory of variational inequalities, visco-elasticity, or plasticity. But he is perhaps even more remembered for the manifold landmark contributions he made to the research of nonlinear partial differential equations, notably by recognizing the efficiency of compactness, monotony, regularization, and penalty methods for their analysis.

With an incredible intuition, Jacques-Louis Lions foresaw very early the advantage of Galerkin methods, for instance, how the finite element method exceeds the more traditional finite-difference methods. In so doing, he was highly instrumental in the creation of a very powerful school of numerical analysts “without frontiers” (across national boundaries), who made many extraordinary breakthroughs to the theoretical understanding as well as to the practical implementation of a wide array of methods for approximating the solutions of partial differential equations. He also made pioneering contributions to the analysis of problems with small parameters and, more generally, of singular perturbation problems.

But his ever-favorite subject was control theory, where, as far back as in 1958, he made milestone advances in the extension of optimal control to systems governed by partial differential equations. One highlight of his contributions to this field was the prestigious “John von Neumann Lecture” that he gave at the SIAM Congress in Boston in 1986, where he laid the foundations of his well-known “HUM method”.

One can only be impressed by his immense works, for the quality, diversity, or novelty of the mathematics used, and for his permanent quest for new applications that had previously been believed to be inaccessible.

Jacques-Louis Lions was a visionary, who quickly understood that the availability of ever-increasing computational power would revolutionize the modeling of numerous phenomena, provided however that the required mathematics were simultaneously created and developed. This is the essence of his immense scientific heritage.

Jacques-Louis Lions justly received numerous honors. In particular, he was a member of twenty-two academies, which included the most prestigious ones, such as the Royal Society, the USSR Academy of Sciences, the National Academy of Sciences of the USA, the French Academy of Sciences, the Third World Academy of Sciences, the Accademia Nazionale dei Lincei, and the Chinese Academy of Sciences. He was also awarded such highly prestigious prizes as the John von Neumann Prize, the Lagrange Prize of the ICIAM, and the Japan Prize.

It is to honor the scientific heritage of Jacques-Louis Lions that an “International Conference on Partial Differential Equations: Theory, Control and Approximation” was organized and held at Fudan University in Shanghai from May 28th to June 1st, 2012. This conference brought together experts from all over the world, whose talks covered the fields of research that Jacques-Louis Lions created or contributed so much to create. This book gathers some of the most representative contributions to the Conference, which have been and will be separately published in Chinese Annals of Mathematics in 2013 and 2014. We thank Ms. Wei Wu of the Editorial Board Office of Chinese Annals of Mathematics for her enthusiastic and effective work in editing this collection of papers.

All those who approached Jacques-Louis Lions will cherish the memory of his warm personality, the vision that he so well conveyed, and his profound intelligence.

Philippe G. Ciarlet  
Tatsien Li  
Yvon Maday

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