

BOLYAI SOCIETY
MATHEMATICAL STUDIES

25

BOLYAI SOCIETY MATHEMATICAL STUDIES

Editor-in-Chief:
Gábor Fejes Tóth

Series Editor:
Dezső Miklós

Publication Board:

Gyula O. H. Katona · László Lovász · Péter Pál Pálffy
András Recski · András Stipsicz · Domokos Szász

1. **Combinatorics, Paul Erdős is Eighty, Vol. 1**
D. Miklós, V.T. Sós, T. Szőnyi (Eds.)
2. **Combinatorics, Paul Erdős is Eighty, Vol. 2**
D. Miklós, V.T. Sós, T. Szőnyi (Eds.)
3. **Extremal Problems for Finite Sets**
P. Frankl, Z. Füredi, G. O. H. Katona, D. Miklós (Eds.)
4. **Topology with Applications**
A. Császár (Ed.)
5. **Approximation Theory and Function Series**
P. Vértesi, L. Leindler, Sz. Révész, J. Szabados, V. Totik (Eds.)
6. **Intuitive Geometry**
I. Bárány, K. Böröczky (Eds.)
7. **Graph Theory and Combinatorial Biology**
L. Lovász, A. Gyárfás, G. Katona, A. Recski (Eds.)
8. **Low Dimensional Topology**
K. Böröczky, Jr., W. Neumann, A. Stipsicz (Eds.)
9. **Random Walks**
P. Révész, B. Tóth (Eds.)
10. **Contemporary Combinatorics**
B. Bollobás (Ed.)
11. **Paul Erdős and His Mathematics I+II**
G. Halász, L. Lovász, M. Simonovits, V. T. Sós (Eds.)
12. **Higher Dimensional Varieties and Rational Points**
K. Böröczky, Jr., J. Kollár, T. Szamuely (Eds.)
13. **Surgery on Contact 3-Manifolds and Stein Surfaces**
B. Ozbagci, A. I. Stipsicz
14. **A Panorama of Hungarian Mathematics in the Twentieth Century, Vol. 1**
J. Horváth (Ed.)
15. **More Sets, Graphs and Numbers**
E. Győri, G. O. H. Katona, L. Lovász (Eds.)
16. **Entropy, Search, Complexity**
I. Csizsár, G. O. H. Katona, G. Tardos (Eds.)
17. **Horizons of Combinatorics**
E. Győri, G. O. H. Katona, L. Lovász (Eds.)
18. **Handbook of Large-Scale Random Networks**
B. Bollobás, R. Kozma, D. Miklós (Eds.)
19. **Building Bridges**
M. Grötschel, G. O. H. Katona (Eds.)
20. **Fete of Combinatorics and Computer Science**
G. O. H. Katona, A. Schrijver, T. Szőnyi (Eds.)
21. **An Irregular Mind, Szemerédi is 70**
I. Bárány, J. Solymosi (Eds.)
22. **Cylindric-like Algebras and Algebraic Logic**
H. Andréka, M. Ferenczi, I. Németi (Eds.)
23. **Deformations of Surface Singularities**
A. Némethi, Á. Szilárd (Eds.)
24. **Geometry (Intuitive, Discrete and Convex), A Tribute to László Fejes Tóth**
I. Bárány, K. Böröczky, Jr., G. Fejes Tóth, J. Pach (Eds.)

László Lovász
Imre Z. Ruzsa
Vera T. Sós
(Eds.)

Erdős Centennial



Springer



JÁNOS BOLYAI MATHEMATICAL SOCIETY

László Lovász
Eötvös Lóránd University
Department of Computer Science
Pázmány P. sétány 1/c
Budapest 1117
Hungary
e-mail: lovasz@cs.elte.hu

Managing Editor:
Dömötör Pálvölgyi
Eötvös Lóránd University
Department of Computer Science
Pázmány P. sétány 1/c
Budapest 1117
Hungary
e-mail: dom@cs.elte.hu

Imre Z. Ruzsa
Alfréd Rényi Institute of Mathematics
Hungarian Academy of Sciences
Reáltanoda u. 13–15
Budapest 1053
Hungary
e-mail: imre.z.ruzsa@renyi.mta.hu

Vera T. Sós
Alfréd Rényi Institute of Mathematics
Hungarian Academy of Sciences
Reáltanoda u. 13–15
Budapest 1053
Hungary
e-mail: t.sos.vera@renyi.mta.hu

Mathematics Subject Classification (2010):
03EXX, 05-XX, 11BXX, 11KXX, 11MXX, 11PXX, 20BXX, 28-XX, 41-XX, 60BXX

Library of Congress Control Number: 2013941387

ISSN 1217-4696
ISBN 978-3-642-39285-6 Springer Berlin Heidelberg New York
ISBN 978-963-9453-18-0 János Bolyai Mathematical Society, Budapest

This work is subject to copyright. All rights are reserved, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, reuse of illustrations, recitation, broadcasting, reproduction on microfilm or in any other way, and storage in data banks. Duplication of this publication or parts thereof is permitted only under the provisions of the German Copyright Law of September 9, 1965, in its current version, and permission for use must always be obtained from Springer. Violations are liable for prosecution under the German Copyright Law.

Springer is a part of Springer Science+Business Media
springer.com

© 2013 János Bolyai Mathematical Society and Springer-Verlag

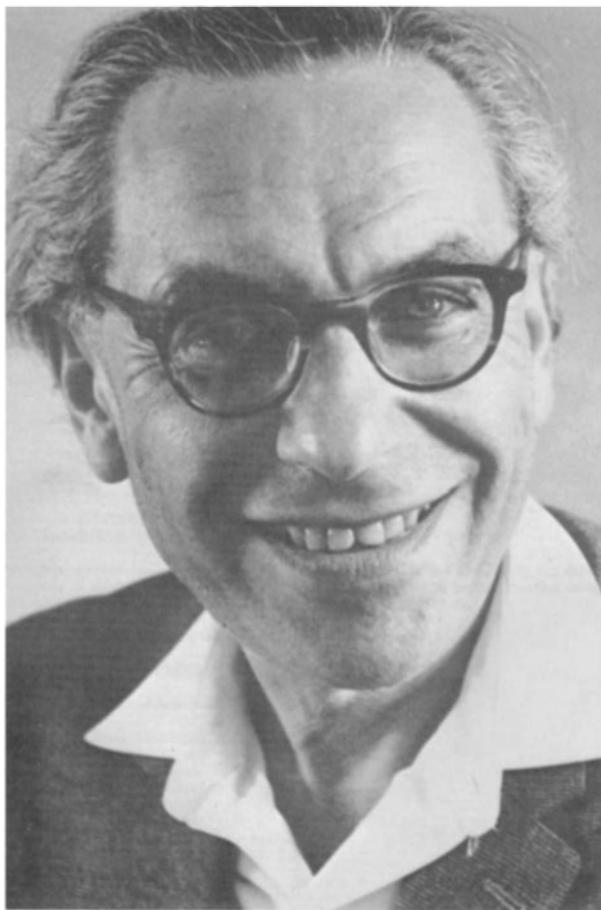
The use of general descriptive names, registered names, trademarks, etc. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant protective laws and regulations and therefore free for general use.

Cover photo of Paul Erdős is courtesy of J. Schönheim
Cover design: WMXDesign GmbH, Heidelberg
Printed on acid-free paper 44/3142/db - 5 4 3 2 1 0

CONTENTS

CONTENTS	5
PREFACE	9
ALON, N.: Paul Erdős and Probabilistic Reasoning	11
BENJAMINI, I.: Euclidean vs. Graph Metric	35
BOLLOBÁS, B. and RIORDAN, O.: The Phase Transition in the Erdős–Rényi Random Graph Process	59
BOURGAIN, J.: Around the Sum-product Phenomenon	111
BREUILLARD, E., GREEN, B. and TAO, T.: Small Doubling in Groups	129
DIAMOND, H. G.: Erdős and Multiplicative Number Theory	153
FÜREDI, Z. AND SIMONOVITS, M.: The History of Degenerate (Bipartite) Extremal Graph Problems	169
GOWERS, W. T.: Erdős and Arithmetic Progressions	265
GRAHAM, R. L.: Paul Erdős and Egyptian Fractions	289
GYÖRY, K.: Perfect Powers in Products with Consecutive Terms from Arithmetic Progressions, II	311
KOMJÁTH, P.: Erdős’s Work on Infinite Graphs	325
KUNEN, K.: The Impact of Paul Erdős on Set Theory	347
MAULDIN, R. D.: Some Problems and Ideas of Erdős in Analysis and Geometry	365
MONTGOMERY, H. L.: L^2 Majorant Principles	377
NEŠETŘIL, J.: A Combinatorial Classic – Sparse Graphs with High Chromatic Number	383
NGUYEN, H. H. and VU, V. H.: Small Ball Probability, Inverse Theorems, and Applications	409
PACH, J.: The Beginnings of Geometric Graph Theory	465
PINTZ, J.: Paul Erdős and the Difference of Primes	485
POLLACK, P. and POMERANCE, C.: Paul Erdős and the Rise of Statistical Thinking in Elementary Number Theory	515

RÖDL, V. and SCHACHT, M.: Extremal Results in Random Graphs	535
SCHINZEL, A.: Erdős's Work on the Sum of Divisors Function and on Euler's Function	585
SHALEV, A.: Some Results and Problems in the Theory of Word Maps	611
TENENBAUM, G.: Some of Erdős' Unconventional Problems in Number Theory, Thirty-four Years Later	651
TOTIK, V.: Erdős on Polynomials	683
VÉRTESI, P.: Paul Erdős and Interpolation: Problems, Results, New Developments	711



Paul Erdős

Paul Erdős 1913–1996

PREFACE

Paul Erdős was one of the most influential mathematicians of the twentieth century. His work in number theory, combinatorics, set theory, and other branches of mathematics has determined the development in large areas of these fields. His name is forever attached to combinatorial and additive number theory, combinatorial geometry, extremal graph and hypergraph theory, random graphs, and the probabilistic method. His contributions to set theory, the theory of primes, analysis, probability, and other classical areas in mathematics are also fundamental.

Paul Erdős passed away in 1996. Three years later, a conference was organized in Budapest to survey his work, his contributions to mathematics, and the far-reaching impact of his work on many branches of mathematics. A 2-volume collection of papers, “Paul Erdős and his Mathematics” (János Bolyai Mathematical Society and Springer-Verlag 2002), was also published, which contained papers about his life, surveys of areas which he initiated or contributed to, and personal reminiscences by his friends and collaborators.

We feel that in 2013, on the 100th anniversary of his birth, it was time to have another look on the long-term impact of his work. We are organizing another conference devoted to his mathematics. This volume (which is not the Proceedings of this conference, but of course having the similar goals) undertakes the almost impossible task to describe the ways in which problems raised by him and topics initiated by him (indeed, whole branches of mathematics) continue to flourish.

Written by outstanding researchers in these areas, the papers in this volume include extensive surveys of classical results as well as of new developments. It would be even more hopeless to be comprehensive than in 1999, but we hope that this volume, as well as the lectures at the conference, will give a glimpse into how his mind was working, and a feeling for his tremendous influence on modern mathematics.

The interested reader should also consult the home page of the conference (<http://www.renyi.hu/erdos100>), which contains more material, including the program and abstracts of posters submitted to the conference. We plan that recordings of plenary talks will also be made available. The Paul Erdős page (http://www.renyi.hu/~p_erdos) contains scanned copies of most Erdős papers, along with many photos and a lot of other material.

Our thanks are due to Dömötör Pálvölgyi for his very careful and efficient work as managing editor of this volume, to Dezső Miklós for organizing the production, and to Ildikó Miklós for the expert production of the \LaTeX files.

Budapest, May 2013

László Lovász
Imre Z. Ruzsa
Vera T. Sós