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Hysteresis Phenomena in Biology

 Springer

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*We should be careful to get out of an
experience only the wisdom that is
in it—and stop there.*

Mark Twain

Preface

For over a century, nonlinear phenomena of hysteresis type have been ubiquitous in different areas of science and technology from the physics of materials to the meta-physics of the human behavior. However, mathematical approaches to investigate biological processes with hysteresis nonlinearities have a much shorter history. Although the idea of modeling chemical and biological systems with multiple steady states and state transition dynamics (switches) goes back to Lotka (1924) and Volterra (1931), it was by the early 1990s that a sufficient number of discovered molecular mechanisms with bistable dynamical behavior gave rise to mathematical investigation of biological switches and hysteresis phenomena and a steady increase in the number of publications from 99 articles in 1990 to 562 in the year 2012 (www.ncbi.nlm.nih.gov/pubmed). To date, mathematical models of hysteresis are used to describe a variety of biological phenomena including the dynamics of metabolic networks, conformational changes of transmembrane proteins, and the biomechanical properties of organs such as the lungs and the eye.

Despite the growing interest in this field of research, the access to its theoretical foundations is limited to expert mathematicians, physicists, and engineers. While the seminal monographs by Krasnosel'skii and Pokrovskii (1989), Visintin (1994), Brokate and Sprekels (1996), and Mayergoyz (2003) enlighten the mathematical theory of hysteresis with a view toward physical and engineering problems, less attention was paid to its applications in biology and chemistry. To close this gap, this monograph is written by an interdisciplinary researcher for graduate students and interdisciplinary researchers. It is an introductory graduate-level textbook on the fundamental concepts of hysteresis with a view toward biological applications, for students and researchers with an interest in mathematical and computational biology who already have a solid acquaintance with dynamical systems, ordinary differential equations, and linear functional analysis.

This book comprises two parts and is mostly based on my lectures on dynamical systems and on mathematical models of hysteresis in the University of Heidelberg. In the first part (**Chaps. 1–3**), we follow the footsteps of Krasnosel'skii and Pokrovskii (1989) and systematically introduce the mathematics of hysteresis. **Chapter 1** is an introduction in the concept of hysteresis in light of biological processes with a particular focus on biological switches and bistable phenomena. In analogy to the spectral decomposition of self-adjoint operators in the functional analysis, all Preisach models are superpositions of elementary nonlinear hysteresis

operators (rectangular loops resembling bistable systems) called hysterons. This concept might appear as a very intuitive one for biologists and therefore builds the theoretical basis of this monograph. Thus, [Chap. 2](#) recapitulates the fundamental definitions and ideas from nonlinear analysis on the stability and bifurcation theory of dynamical systems and prepares the transition toward hysteresis models. The global aim of this chapter is to provide the necessary definitions and theorems with an emphasis directed toward hysteresis so that the interested reader can focus on the essential aspects. Particularly, we will recapitulate the Lyapunov stability and Andronov's structural stability concepts, which are also of general interest for mathematical modeling with ordinary differential equations in biology. The information in this chapter is mainly compiled from the monographs on the theory of dynamical systems and ordinary differential equations by Verhulst (2008) and Perko (2008) and the experienced reader may skip this chapter. [Chapter 3](#) is primarily concerned with Preisach models of hysteresis, their identification problems, and their numerical simulations. In particular, we will deal with the classical Preisach theory of hysteresis for continuous and periodic input functions, the parallel and sequential superpositions of hysterons, and the impact of perturbations of input function on the systems. Furthermore, in compliance with Mayergoyz (2003) we will discuss a strategy to identify the weights of Preisach operators from experimental data and we discuss an approach using first-order transition curves for numerical implementation of hysteresis models. On the basis of these mathematical theories, the second part of the book ([Chap. 4](#)) demonstrates a few examples of applications of hysteresis theory in the modeling and numerical simulation of biological and chemical systems, particularly the development of mathematical models of bistability and hysteresis-like behavior in different biological disciplines.

Based on the large number of relevant publications, no attempt has been made to exhaust the list of references but to introduce those with the most recognizable impact in their fields. This imperfection is reflected throughout this monograph, as I have made no attempt at all to include all available information. Instead, my focus was to utilize the interested reader with a systematic introduction to a very complicated mathematical theory and reveal its potential for modeling biological and chemical processes. My first contact with mathematical models of hysteresis was during my conversations with Professor Willi Jäger as I was a Ph.D. student in his workgroup some years ago. I was fascinated by its nonlinear nature and was ambitious to study and apply it on every problem that I was working on. However, my impression was that the mathematics of hysteresis and its applications that were of interest for me, namely neuroscientific problems, were still disconnected and to use the models for biological problems, one had to go through the entire mathematical theories. My feeling was further corroborated by the feedbacks that I received from the students and postdocs who attended my lecture on mathematical models of hysteresis in the institute for applied mathematics of the University of Heidelberg, which motivated me to rewrite my lecture notes with a constant view toward the applications. I was very fortunate that my old friend Dr. J. P. Schmidt (Springer) approached me to publish those notes as an introductory book. I would

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References

- Brokate M, Sprekels J (1996) Hysteresis and phase transitions. Springer, New York
Krasnosel'skii MA, Pokrovskii AV (1989) Systems with hysteresis. Springer, Heidelberg
Lotka AJ (1924) Elements of physical biology. Dover, New York
Mayergoyz ID (2003) Mathematical models of hysteresis and their applications. Elsevier, New York
Perko L (2008) Differential equations and dynamical systems. Springer, Heidelberg
Verhulst F (2008) Nonlinear differential equations and dynamical systems. Springer, Berlin Heidelberg
Visintin A (1994) Differential models of hysteresis. Springer, Berlin
Volterra V (1931) Lecons sur la theorie de la lutte pour la vie. Gauthier-Villars, Paris

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