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Preface

The organization of the 17th International Conference on Discrete Geometry for Computer Imagery (DGCI 2013) has been a rewarding experience for our Andalusian research group, Combinatorial Image Analysis (CIMAgroup). As organizers, we are pleased with the participation of many researchers from all around the world, taking into account the financial difficulties of our times. Indeed, submissions from 26 different countries confirm the international status of the conference.

This collection documents the contributions presented at DGCI 2013, which focus on geometric transforms, discrete and combinatorial tools for image segmentation and analysis, discrete and combinatorial topology, discrete shape representation, recognition and analysis, models for discrete geometry, morphological analysis, and discrete tomography.

Following a peer-reviewing process by at least two qualified reviewers, 34 papers were accepted, out of 56 submissions. Altogether, 22 papers were scheduled for oral presentation in single-track sessions, and 12 papers were presented as posters.

It has been a great honor for us to count on the participation of three internationally well-known researchers as invited speakers: Herbert Edelsbrunner (Professor at the Institute of Science and Technology, Vienna University, Austria), Francisco Escolano (Associate Professor at the University of Alicante, Spain), and Konrad Polthier (MATHEON-Professor and Director of the Mathematical Geometry Processing group at Freie Universität Berlin, Germany).

We would like to express our gratitude to the Reviewing and Program Committee members for their valuable comments, which enabled the authors to improve the quality of their contributions. We are also grateful to the Steering Committee for giving us the chance to organize this event and especially to David Coeurjolly for his support and helpful advice.

DGCI 2013 has been supported by the International Association of Pattern Recognition (IAPR). DGCI conferences are the main events associated with the Technical Committee on Discrete Geometry IAPR-TC18. This conference could not have been organized without our sponsoring institutions: University of Seville (Vice-rectorate for Research, Vice-rectorate of Internationalization, the Mathematics Institute IMUS, the Research and Teaching Foundation FIDETIA, Applied Math-I Department), Spanish Ministry of Economy and Competitiveness (Project MTM2012-32706), and European Science Foundation (ACAT program). We are also grateful to the School of Computer Engineering at the University of Seville, for hosting this event and providing all the necessary facilities.
Finally, our special thanks go to the local Organizing Committee for their valuable work and to all the participants attending the conference, who made this event a success.

March 2013

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Abstract. In this paper we explore how to quantify the complexity of discrete objects (images, meshes, networks) which can be encoded in terms of graphs, digraphs or hypergraphs. Herein we present our Heat Flow-Thermodynamic Depth approach which combines ingredients of spectral graph theory, information theory and information projection. We illustrate the approach with several applications: image exploration (image complexity), mesh regularization and selection of optimal map functions in Extended Reeb Graphs (graph and digraph complexity) and structural categorization (hypergraph complexity).

1 Randomness vs. Statistical Complexity

Given a discrete mathematical object (image, mesh, network) which can be encoded by a graph, digraph or hypergraph, the quantification of its intrinsic complexity plays a key role in understanding the underpinning structural principles shaping it. Such principles include: the information content of the encoding, the set of constraints over the information flowing through it, and the combinatorial exploration of the hypothesis that best explain its genesis. Information content may be posed in terms of estimating a given entropy. Information flow constraints may be discovered by probing the structure through heat kernels and wave equations. Generative hypothesis may rely on depth-based representations like logical depth or thermodynamic depth, hierarchical representations like grammars and compositional models, or dynamic rules like preferential attachment. The weight of each of the latter principles in the characterization of the encoding defines a computable complexity measure for it. For instance, strategies considering information content through Shannon or cross entropy are referred to as randomness complexity measures for they quantify the degree of disorganization (see a recent survey on graph entropy measures in [1]) . On the other hand, methods relying on spectral measures, like the von Neumann entropy [2], or on Dirichlet sums, like Estrada’s heterogeneity index are [3] more focused on quantifying the degree of regularity of the structure. These are good examples of the so called statistical complexity measures that vanished both for completely regular and completely random structures. Thermodynamic depth (TD) is a physics based approach [4] also belonging to the latter category. When dealing with graphs, termodynamic depth aims to quantify how hard is to build a given graph (the macroscopic state) from scratch (microscopic states): if there
is a little uncertainty about the process and all the possible causal trajectories have a narrow variability, then the graph is narrow (simple); otherwise, when historical uncertainty arises and many causal trajectories have been excluded for reaching a given structural design, then the graph is deep (complex). Therefore, TD is purely based on the genesis principle. In this talk we present an statistical complexity measure which is focused on the connection between the second principle (information constraints) and the third one by exploring the connection between heat diffusion and TD. The underlying idea is that structure may impose constraints on heat diffusion. For instance, a complete graph must have zero complexity since there are no diffusion constraints. On the other hand, a linear graph imposes hard constraints. This occurs for both undirected and directed graphs. The combination of heat flow and TD breaks isospectrallity. In [5] we present the main ingredients of the theory, embed both the von Neumann entropy and the Estrada’s heterogeneity index in TD and show that the embedding of heat flow in TD outperforms the latter embeddings in terms of predicting the phylogeny of Protein-Protein Interaction (PPI) networks of several phyla of bacteria. In [6] we extended the so called Heat Flow-TD complexity to digraphs and compute the surface complexity of many European languages. In the following section we summarize some of the applications appealing to the DGCI community which are explored in the talk.

2 Applications of Structural Complexity

2.1 The Complexity of Images

We commence by motivating the need of measuring graph complexity beyond regularization or the minimization of description length. In [7] we propose an information-theoretic approach to understand eye-movement patterns, in relation to the task performed and image complexity. We analyze the distributions and amplitudes of eye-movement saccades, performed across two different image-viewing tasks: free viewing and visual search. Our working hypothesis is that the complexity of image information and task demands should interact. This should be reflected in the Markovian pattern of short and long saccades. We compute high-order Markovian models of performing a large saccade after many short ones and also propose a novel method for quantifying image complexity. The analysis of the interaction between high-order Markovianity, task and image complexity supports our hypothesis. Image complexity is measured in terms of computing the stationary distribution of a random walk in a grid-like structure whose nodes are image regions characterized by the responses of several filters. The edges in the grid are attributed by the mutual information between adjacent nodes. Given the stationary distribution we compute its Shannon entropy. Therefore, in this application we exploit a randomness complexity measure. To the best of our knowledge this is the first scientific connection between complexity quantification, perceptual tasks and image structure.
**Fig. 1.** SHREC complexities. TD complexity for each object in each class. For each class the dashed horizontal line and the number indicates the median TD complexity. Typical shapes (in classes with low complexity) and complex shapes (in classes with peaks).

### 2.2 The Complexity of Reeb Graphs

It is well known that the basic idea of Reeb graphs is to obtain information concerning the topology of a manifold \( M \) from information related to the critical points of a real function \( f \). The Reeb graph produces a structure whose nodes are associated with the critical points of \( f \). Here we follow the computational approach in [8], where a discrete counterpart of Reeb graphs, referred to as the **Extended Reeb Graph** (ERG), is defined for triangle meshes representing surfaces in \( \mathbb{R}^3 \). The basic idea underpinning ERG is to provide a region-based characterization of surfaces, rather than a point-oriented characterization. This is done by replacing the notion and role of critical points with that of **critical areas**, and the analysis of the behaviour of the level sets with the analysis of the behaviour of surface stripes, defined by partitioning the co-domain of \( f \) into a finite set of intervals. We consider in more detail a finite number of level sets of \( f \), which divide the surface into a set of regions. Each region is classified as a **regular** or a **critical area** according to the number and the value of \( f \) along its boundary components. Critical areas are classified as **maximum**, **minimum** and **saddle** areas. A node in the graph is associated with each critical area. Then arcs between nodes are detected through an expansion process of the critical areas, by tracking the evolution of the level sets. A fundamental property of ERGs is their parametric nature with respect to the mapping function \( f \): different choices of \( f \) produce different graphs. Also, the graphs inherit the invariance properties, if any, of the underlying mapping function. The mapping function...
has to be selected according to the invariance and shape properties required for the task at hand. For instance, the analysis of the SHREC database from the point of view of barycenter Reeb graphs is summarized in Fig. 1. In this figure we represent the thermodynamic depth complexities of the 300 models and their variation among classes (median). Some classes are more heterogeneous than others. Therefore, thermodynamic depth of graphs seems to be a MDL (Minimum Description Length)-like measure of the mapping functions used for extracting Reeb graphs from 3D shapes. We will analyze several mapping functions and classify them in terms of intraclass variability (stability). This analysis concerns both undirected and directed graphs and both attributed and non-attributed ones. The spectral graph theory machinery developed for capturing the information flowing through Reeb graphs relies on the analysis of the combinatorial Laplacian and the evolution of heat kernels and quantum walks. Such elements are combined with the computation of node histories (TD) and also with Bregman divergences and information projection in order to quantify complexity.

2.3 The Complexity of Hypergraphs

Initial experiments with PPI networks show that complexity can be very helpful for structural classification purposes [9]. Recently, depth-based representations have been extended to characterize hypergraphs resulting in a high performance structural classifiers. Hypergraphs also allow us to mix several mapping functions for Reeb graphs which will be very helpful in their classification.

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References

Stable Length Estimates of Tube-Like Shapes

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Abstract. Mathematical objects can be measured unambiguously, but not so objects from our physical world. Even the total length of tube-like shapes has its difficulties. We introduce a combination of geometric, probabilistic, and topological methods to design a stable length estimate for tube-like shapes; that is: one that is insensitive to small shape changes.

1 Introduction

The length of a curve in Euclidean space is an elementary geometric concept, and it is well defined provided the curve is not wild. We consider the problem of computing the length of curve- or tube-like shapes, such as root systems of plants. Branching is allowed, but the real difficulty lies in the small but positive thickness, which renders length an undefined concept, at least in the mathematical sense. One may want to construct a 1-dimensional skeleton and take the length of that, but this construction is unstable; see [1, 5]. Instead of stabilizing the skeleton, we aim at estimating the length of a hypothetical skeleton, which we leave unspecified. The difficulty in the related case of a coast line, studied famously by Mandelbrot [12], is the dependence on the resolution to which the curve is being measured. The length diverges as the resolution increases, suggesting the dimension of the coast line be larger than 1.

Noticing the abundance of tube-like shapes in nature and therefore in the sciences, we aim at producing a length estimate that is stable under perturbations of the shape. We believe that this will be useful in the study of geographic structures, including river and road networks, as well as biological and medical structures, including root systems of plants, blood vessels, nerve cells, and more. Our length estimation algorithm combines intuitive geometric ideas with topological methods:

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1. using the formula of Weyl [8, 14, 15], it expresses the length of a tube-like shape by an integral geometric representation of the second Quermassintegral;
2. applying a recent version of the Koksma-Hlawka Theorem [9–11], it approximates the resulting integral with explicit bounds on the integration error;
3. exploiting insights into the persistence diagram of a height function [2, 3, 7], it gives a length estimate that is stable under perturbations.

We implement the algorithm and analyze its performance. Our experiments give clear evidence for the effectivity of the topological method and the stability of the length estimate provided by our algorithm.

2 Tubes and Integral Geometry

We study and extend special cases of the tube formula of Weyl [8, 13, 15]. This formula holds for general smooth submanifolds of a finite-dimensional Euclidean space. The main result of this section is a simple relationship between the geometric properties of a curve and its thickened version in $\mathbb{R}^3$. Letting $r_0$ be the thickening radius, we denote the thickened curve by $M$. We have

$$L = Q_2/\pi;$$

$$L = [Q_2 - k(2\pi - 4)r_0 - 4\pi r_0]/\pi,$$

where $L$ is the length of the curve, which is closed in (1), and has $k \geq 0$ right-angle forks and $k + 2$ tips in (2). For $k = 0$, we have a curve with two endpoints.

To define $Q_2$, we introduce

$$Q_i(M) = c_i \cdot \int_P \chi(M \cap P) dP,$$

called the $i$-th Quermassintegral over the set of $i$-dimensional planes, $P$, in which $c_i$ is a constant independent of $M$. For $i = 2$, we have $c_i = 1$, and $Q_2 = Q_2(M)$ is the total mean curvature of the bounding surface.

3 Quasi Randomness

In order to evaluate the second Quermassintegral, we apply a version of the Koksma-Hlawka Theorem recently proved by Harman [9]. This theorem bounds the integration error,

$$\text{Err}(N, X) = \left| \frac{1}{N} \sum_{j=1}^{N} F(x_j) - \int F(x) \, dx \right|,$$

in which $X = \{x_1, x_2, \ldots, x_N\}$ denotes a finite point set. It separates the contributions of the variation of the function and the distribution properties of the points at which the function is evaluated.
4 Persistence and Stability

We modify the straight length estimation formulas to get stable estimates for tube-like shapes. The tool for this purpose is persistent homology; see e.g. [6]. The main result is an expression of stability for the damped persistence moments of a function:

1 (Damped Stability Theorem for Tubes) Let $\mathbb{M}$ be a tube of radius $r_0$ in $\mathbb{R}^3$, let $f, g : \mathbb{M} \to \mathbb{R}$, and set $C = 4 + \delta$. Then for every dimension $0 \leq p \leq 2$, every direction $u \in \mathbb{S}^2$, and every $\delta > 0$, we have

$$|X_p^C(f) - X_p^C(g)| \leq \text{const} \cdot \frac{L^{1+\delta}}{r_0^{1+\delta}} \cdot \|f - g\|_\infty. \quad (5)$$

Here $X_p^C(f)$ is related to the persistence moments, as we now explain. First, introduce

$$B^k_p(f) = \sum_{A \in \mathcal{U}_p(f)} \text{pers}(A)^k + \sum_{A \in \mathcal{D}_{p+1}(f)} \text{pers}(A)^k, \quad (6)$$

in which the two sums are over the points in the persistence diagrams of the function. Specifically, $\mathcal{U}_p(f)$ consists of all points in the $p$-dimensional diagram whose birth-coordinates are smaller than the death-coordinates, and $\mathcal{D}_{p+1}(f)$ consists of all points in the $(p + 1)$-dimensional diagram whose birth-coordinates are larger than the death-coordinates. We call $B^k_p(f)$ the $k$-th $p$-dimensional persistence moment. For $k = 1$, its significance lies in the relationship to the second Quermassintegral:

$$Q_2(\mathbb{M}) = \frac{1}{2} \int_{u \in \mathbb{S}^2} \left( \sum_{p=0}^{2} (-1)^p B^1_p(f_u) \right) du, \quad (7)$$

where $f_u$ is the height function on $\mathbb{M}$ in the direction $u$. We split $B^k_p(f) = B^k_p(f, r_0^-) + B^k_p(f, r_0^+)$ by collecting the values for $\text{pers}(A) < r_0$ in the first term and values for $\text{pers}(A) \geq r_0$ in the second term. For $C = 4 + \delta$, we set

$$X_p^C(f) = B^1_p(f, r_0^+) + \frac{1}{r_0^{C-1}} \cdot B^C_p(f, r_0^-), \quad (8)$$

$$\bar{Q}_2(\mathbb{M}) = \frac{1}{2} \int_{u \in \mathbb{S}^2} \left( \sum_{p=0}^{2} (-1)^p X_p^C(f_u) \right) du, \quad (9)$$

calling $\bar{Q}_2 = \bar{Q}_2(\mathbb{M})$ the stabilized mean curvature of $\mathbb{M}$ at scale $r_0$. The Damped Stability Theorem now implies a similar expression of stability for the stabilized mean curvature.
5 Computational Experiments

We describe experimental results for the algorithms implementing the mathematical formulas developed in the preceding sections. We test accuracy as well as stability on small datasets, for which the answers are known, and investigate speed of convergence on root system data. We use three different algorithms to compute or approximate the total mean curvature of the boundary of a polytope $M$ in $\mathbb{R}^3$, and to estimate the length of $M$:

- **Discrete Mean Curvature (DMC)**: we compute the total mean curvature as half the sum over all boundary edges of the length times the angle between the two adjacent face normals; see e.g. [4].
- **Plane Sampling (PS)**: we approximate the total mean curvature by summing up the Euler characteristics of the intersections between $M$ and planes sampled in $\mathbb{R}^3$.
- **Direction Sampling (DS)**: we approximate the total mean curvature by summing up the alternating persistence moments of height functions defined by sampled directions on the 2-sphere.

The result of the **DMC** Algorithm is the total mean curvature of $M$ up to machine precision, which we use as the baseline for comparisons. The result of the **PS** Algorithm converges to the total mean curvature, and we get an impression of the speed of convergence from a comparison with the precise measurement. The basic version of the **DS** Algorithm is a reformulation of the **PS** Algorithm, but it offers the opportunity to filter out low-persistence contributions, thus stabilizing the length estimate; see the Damped Stability Theorem of the previous section.

Indeed, the design of the algorithm implementing (9) is the main achievement of this paper. Experimental results comparing the performance of this algorithm with others can be found in the full version of this paper.

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