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Timothy J. Hollowood

# Renormalization Group and Fixed Points

in Quantum Field Theory

 Springer

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# Preface

The purpose of this short monograph is to introduce a powerful way to think about quantum field theories. This conceptual framework is Wilson's version of the renormalization group. The only prerequisites are a basic understanding of QFTs along the lines of a standard introductory course: the Lagrangian formalism and path integral, propagators, Feynman rules, etc.

The discussion begins with the simplest theories of scalar fields and then tackles gauge theories. Finally, theories with supersymmetry are briefly considered because they are a wonderful arena for discussing the renormalization group as there are a few key properties that one can prove exactly. For this reason the last chapter will provide a very basic description of some of the features that SUSY theories have with regard to the renormalization group, although the discussion of SUSY itself will necessarily be very rudimentary.

I apologise in advance to those who have pioneered this subject as I have not attempted to make a comprehensive list of references. The references that are given are intended to point the reader to sources which do have comprehensive lists.

I would like to thank the organizers of BUSSTEPP, Jonathan Evans in Cambridge 2008 and Ian Jack in Liverpool 2009, for providing excellently run summer schools that enabled me to develop my idea to teach QFT with the renormalization group as the central pillar. I would also like to thank Aaron Hiscox, Dan Schofield, and Vlad Vaganov for careful readings of the manuscript and for making some useful suggestions.

Swansea, December 2012

Timothy J. Hollowood

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# Acronyms

IR	Infra Red
QFT	Quantum Field Theory
RG	Renormalization Group
SUSY	Supersymmetric/Supersymmetry
UV	Ultra Violet
VEV	Vacuum Expectation Value