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Gradually-varied Flow Profiles in Open Channels

Analytical Solutions by Using Gaussian Hypergeometric Function

Springer

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To
Dr. Cheng-lung Chen

Preface

Many hydraulic engineering works involve the computation of surface profiles of gradually varied flow (GVF) that is a steady nonuniform flow in an open channel with gradual changes in its water surface elevation. The most widely used methods for computing GVF profiles could be classified into the direct integration methods, step methods (i.e., simple numerical methods), and advanced numerical methods (e.g., the standard fourth order Runge–Kutta method). Numerical solutions of the GVF equation are primarily used in non-prismatic channels. In some prismatic channels, such as artificial channels, the GVF equation can be simplified so as to let the analytical (or semi-analytical) direct integration be applied. The analytical direct-integration method is straightforward and can provide the total length of the profile in a single computation step.

In the direct-integration method, the one-dimensional GVF equation is usually normalized to a simpler expression in advance so as to allow the performance of direct integration. In most cases, the GVF equation is normalized by the normal depth, while in some cases, it is normalized by the critical depth. Many attempts have been made by previous investigators on the direct-integration method to solve the GVF equation. The table of the varied-flow function (VFF) is needed in the Bakhmeteff-Chow procedure to obtain the GVF profiles by using the conventional direct-integration method. The use of the conventional Bakhmeteff-Chow procedure to obtain GVF profiles has a drawback caused by the inconvenience of the use of the VFF-table and the imprecise interpolation of the VFF-values from the VFF table. To overcome the drawback, Dr. C. L. Chen and the author had initiated a self-investigation to look for an alternative method to get analytical solutions of GVF profiles without recourse to the VFF since 2005. Dr. Chen was an excellent hydrologist in the U.S. Geological Survey, and a distinguished professor in the University of Illinois at Urbana-Champaign and in the Utah State University. Unfortunately, Dr. Chen passed away in January 2012, just about the time at which we found that the Gaussian hypergeometric functions (GHF) can be used to analytically solve the GVF equation, and completed two drafts of manuscripts to be submitted to a suitable journal. After some revisions by the author, these two manuscripts were submitted to two journals and finally published, one in the Journal of Hydrology in August 2012, and another in the Hydrology and Earth System Sciences (HESS) in March 2013, respectively.

Professor K. Hutter, the editor of Advances in Geophysical and Environmental Mechanics and Mathematics (AGEM^2), encouraged the author to thoroughly write out the innovative method using the GHF in analytically solving the GVF profiles in the form of a book, when he visited our university in May 2012. The author began to prepare this book in September 2012. This book attempts to thoroughly introduce the innovative procedure to obtain the analytical solution of GVF profiles using the Gaussian hypergeometric functions (GHF) as well as to present the analysis and discussions of the GHF-based solutions. This book is divided into five chapters. Chapter 1 introduces the basic equations for the GVF in open channels. Chapter 2 introduces the conventional direct-integration methods used to analytically solve the GVF equation. Chapter 3 presents the GVF equation normalized by the normal depth, and the application of the GHF on analytically solving the normal-depth-based dimensionless equation for GVF in sustaining channels. The classification and properties of the GHF-based solutions of the GVF profiles are also discussed in this chapter. Chapter 4 is devoted to the GVF equation normalized by the critical depth, and then solves it by using GHF for GVF flows in sustaining and non-sustaining channels. Chapter 5 presents the analysis and classification of the GHF-based solutions of the critical-depth-based dimensionless GVF equation. This chapter also shows that the critical-depth-based GVF solution expressed in terms of the GHF is more useful and versatile than its counterpart normalized by using the normal depth.

The author is especially indebted to Prof. C. L. Chen who introduced the author to the subject of GVF profiles as presented in this book, as well as to the subject of debris flow when the author was a graduate student in the University of California, Berkeley between 1988 and 1992, and when the author was a visiting scholar at the U.S. Geological Survey at Menlo Park, California, in 1999. The author also acknowledges Prof. Y. C. Tai and Miss C. C. Shen for their help in the transformation of the manuscript typed in the form of Microsoft Word to that in the form of LaTex. Thanks are due to Prof. K. Hutter and Dr. A. Siviglia for their reviews and valuable suggestions. The author appreciates the supports and resources provided by the National Cheng Kung University, and the National Science Council, Taiwan. Finally, the author would like to express his sincere thanks to all those who have directly or indirectly helped him in writing this book, and to Springer Verlag and its personnel for their help in the production of the book.

The author sincerely hopes that this book will be a reference book for practical civil or hydraulic engineers when they design hydraulic engineering works as well as for undergraduate and graduate students in the fields of civil, hydraulic, and agricultural engineering.

Taiwan, September 1, 2013 Chyan-Deng Jan

Contents

Acronyms and Symbols

Symbol	Name/Descripition
A	Adverse-slope channel
A1, A2, A3	GVF profiles in adverse-slope channels
A, A_n	Cross-section areas
B	Bottom width
C	Critical-slope channel
C1, C2, C3	GVF profiles in critical-slope channels
C	Factors of the flow resistance
C_k	Coefficient relating K and h
C_z	Coefficient relating Z and h
D	Hydraulic depth
d_0	Diameter of a circular channel
ETF	Elementary transcendental function (s)
GVF	Gradually varied flow
GHF	Gaussian hypergeometric function (s)
F	Froude number
g	Gravitational acceleration
$g(b, z)$	Simplified notation of Gaussian hypergeometric function
H	Horizontal-bed channel
H1, H2, H3	GVF profiles in horizontal-bed channels
H	Total energy
h	Flow depth normal to the channel bed
h_c	Critical depth
h_e	Eddy loss
h_f	Frictional loss
h_n	Normal depth
J	Parameter, $J = N/(N - M + 1)$
K, K_c, K_n	Conveyances of the channel section
K_u	Curvature of a GVF profile
M	Mild-slope channel
M1, M2, M3	GVF profiles in mild-slope channels
M	Hydraulic exponent for critical-flow computation
m	Exponent used in the power law of the wall
N	Hydraulic exponent for uniform-flow computation

n	Manning's roughness coefficient
P	Wetted perimeter of cross-section
p	Exponent relating to hydraulic radius
Q	Flow discharged
q	Exponent relating to slopes
R, R_n	Hydraulic radiuses of flow
S	Steep-slope channel
S1, S2, S3	GVF profiles in steep-slope channels
S_0	Slope of channel bed $(= \sin\theta)$
S_{0_*}	$\tan\theta$
S_c	Critical slope
S_{c_*}	$S_c/\cos\theta$
S_f	Energy slope
T, T_c	Top widths of the flow cross-section
u	Dimensionless flow depth $(= h/h_n)$
VFF	Varied-flow function
V, V_n	Mean velocities of flow
v	Dimensionless flow depth $(\upsilon = h/h_c$ or $= u^{N/J})$
w	The reciprocal of u or λv
x	Longitudinal coordinate along the channel bed
x_*	Dimensionless longitudinal coordinate $(= xS_{0_*}/h_n)$
$x_\sharp$	Dimensionless longitudinal coordinate $(= xS_{c_*}/h_c)$
y	Flow depth normal to the datum
Z, Z_c	Section factors
z	Channel side slope, or function variable
z_b	Elevation of channel bed above the datum
α	Energy coefficient
β	Momentum coefficient
θ	Inclined angle of channel bed
λ	Parameter $(= h_c//h_n)$
ψ	Parameter
$\Gamma(a)$	Gamma function