

# Differential-Algebraic Equations Forum

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# Differential-Algebraic Equations Forum

The series “Differential-Algebraic Equations Forum” is concerned with analytical, algebraic, control theoretic and numerical aspects of differential algebraic equations (DAEs) as well as their applications in science and engineering. It is aimed to contain survey and mathematically rigorous articles, research monographs and textbooks. Proposals are assigned to an Associate Editor, who recommends publication on the basis of a detailed and careful evaluation by at least two referees. The appraisals will be based on the substance and quality of the exposition.

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Achim Ilchmann • Timo Reis

Editors

# Surveys in Differential-Algebraic Equations I

 Springer

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# Preface

We are pleased to present the first of three volumes of survey articles in various fields of differential-algebraic equations (DAEs). In the last two decades, there has been a substantial research activity in the theory, applications, and computations of DAEs; our aim is to give an almost complete picture of these latest developments.

What are DAEs? They certainly belong to differential equations, but the terminology is not clear. In their most general form, DAEs are implicit differential equations. However, this is still too wide and in view of linearizations and the fact that most research is on linear DAEs, one uses the more narrow notion of differential-algebraic systems. This fact is reflected in the Mathematics Subject Classification (MSC 2010), which is a taxonomy on a first, second, and third level (2-, 3-, and 5-digit class, respectively). DAEs are mentioned twice on level three: *34 Ordinary differential equations*, *34A General theory*, *34A09 Implicit equations, differential-algebraic equations*, and *65 Numerical analysis*, *65L Ordinary differential equations*, *65L80 Methods for differential-algebraic equations*.

What is the history of DAEs? Although DAEs can be traced back earlier, it was not until the 1960s that mathematicians and engineers started to thoroughly study computational issues, mathematical theory, and applications of DAEs. There are many relationships with mathematical disciplines such as differential geometry, algebra, functional analysis, numerical analysis, stochastics, and control theory, to mention but a few; and there are extensive applications in electric circuit theory, chemical processes, constrained mechanics, as well as in economics. In addition to the intrinsic mathematical interest, there are two fundamental reasons for these advances: first, automatic modeling, which results in large dimensional DAEs, and second the advancement of computers and hence the feasibility of solving problems numerically. In quantitative terms, this development has led to more than 1500 journal and conference papers on DAEs each year.

Is a level two rank, instead of the current level three, for DAEs appropriate? The MSC tries to rank the different levels hierarchically. However, terminological unities for different fields, so that they can be accurately separated from each other, do not necessarily exist. Moreover, fields and their importance vary in time: new fields arise, others become less important. One could imagine that DAEs are equally im-

portant as, for example, *34B Boundary value problems*, *34G Differential equations in abstract spaces*, *34K Functional-differential and differential-difference equations*, *34L Ordinary differential operators*, to name but a few within the 34 ODEs class.

The immense number of papers on DAEs is certainly not a sufficient reason for any taxonomy, and the underlying methods in DAEs are very distinct: differential geometry, distributions, and linear algebra. But today's changing importance and relevance of DAEs have been shown by about ten research monographs in fields of DAEs in the last decade and, most importantly, recently the first textbooks on the mathematical theory of DAEs have been written. This may indicate a turning point: DAEs are becoming a field in their own right, beside other fields in ordinary differential equations.

The collection of survey articles in DAEs presented in the upcoming three volumes will include the topics

- Linear systems
- Nonlinear systems
- Solution theory
- Stability theory
- Control theory
- Model reduction
- Analytical methods
- Differential geometric methods
- Algebraic methods
- Numerical methods
- Coupled problems with partial differential equations
- Stochastic DAEs
- Chemical engineering
- Circuit modelling
- Mechanical engineering

This may show the depth and width of the recent progress in differential-algebraic equations and will possibly underpin the fact that differential-algebraic equations are in a state where they are no longer only a collection of results on the same topic, but a field within the class of ordinary differential equations.

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