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Quantum Ising Phases and Transitions in Transverse Ising Models

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Preface

Quantum phase transitions, driven by quantum fluctuations, have got intriguing features with potentially new application possibilities including those in quantum computations. A significant amount of research in physics today is therefore directed towards the investigations on the nature and behaviour of such quantum phases and transitions in cooperatively interacting many-body quantum systems. Major advances have been made in both theoretical and experimental studies on such systems.

For modelling purposes, although the Heisenberg model and its variants were introduced much earlier (in 1920s), most of the innovative and successful researches today in this field have been obtained by employing, or comparing with, the results of quantum, or transverse field, Ising models (introduced in 1960s). This is because of the advantage in the separability of the cooperative interaction from the (tunable) transverse field or tunnelling term in the Hamiltonian of the transverse field Ising model (in contrast to that in the Heisenberg and other models where the cooperativity and non-commutability are intertwined). Also, a number of condensed matter systems can be modelled accurately by such transverse field Ising models. Because of these, many of the intriguing features observed in the statics or dynamics of quantum phase transitions are mostly limited today to either the theoretical investigations in such quantum Ising models or to the experimental results which can be checked and compared for such models!

This book introduces these quantum Ising models and their theoretical analysis, including numerical ones, at length. Attempts have been made to bring the reader to the research frontiers today.

In an earlier incarnation of this book, it was published in the Lecture Notes in Physics Series of Springer with the same title in 1996 and was authored by Bikas K. Chakrabarti, Amit Dutta & Parangama Sen. Since then, many important developments have taken place, in particular in the dynamic studies like in quantum quenching, annealing etc. Also, the book went out of print rather quickly and Dr. Christian Caron, Executive Publishing Editor of the Springer, requested several times to bring out a second updated edition of the book. However, because of the prolonged nature of the involvements in the respective researches, both Profs. Dutta and Sen could

not agree to join the effort. We therefore decided, with their kind consent, to revise this book thoroughly and upgrade it extensively. The resulting book, though seeded in the earlier one, is also considerably modified and upgraded. Indeed, when this revised book manuscript was given by Dr. Caron to one of their reviewers for an opinion, the reviewer wrote "... authors present an excellent overview on the state of the art of Quantum Ising Phases under various situations ... this reviewer is highly impressed about the excellent quality, the comprehensive and thorough presentation of the material and its expert discussion ... this book is a masterpiece which belongs to the shelves (if not on the desks!) of all researchers working in quantum phase transitions and their equilibrium properties and non-equilibrium dynamics ... authors possess a deep knowledge of the field and are able to account for recent modern developments ... much of the technical (but important) details have wisely been organized by deferring such material to concise appendices at the end of the corresponding chapter ... I have only minor suggestions for improving this 'gem' further ...". These words from the anonymous reviewer clearly offer us *a posteriori* justification of our renewed effort to revise and upgrade the book!

We are also extremely happy to receive many helpful comments and suggestions on the draft version of this book from Profs. Subinay Dasgupta, Amit Dutta and Parongama Sen.

We hope, the book will be useful to the young researchers in exploring this exciting field.

Sagamihara, Japan
Sapporo, Japan
Kolkata, India

Sei Suzuki
Jun-ichi Inoue
Bikas K. Chakrabarti

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