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# Upper Bound Limit Load Solutions for Welded Joints with Cracks

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# Preface

This monograph concerns with application of the upper bound theorem to finding the limit load for welded structures including structures with cracks. The presentation of the introductory material and the theoretical developments appear in a text of six chapters. The topics chosen are primarily of interest to engineers as postgraduates and practitioners but they should also serve to capture a readership from among applied mathematicians. The monograph provides both a collection of limit load solutions for welded structures and a description of general approaches to finding the limit load for a class of structures. Many solutions are represented by formulae. Such solutions are immediately ready for practical use. Other solutions are illustrated by diagrams. These diagrams demonstrate most important tendencies in solutions behavior. It is however evident that they cannot be used for practical calculation of the limit load. Therefore, most of such solutions are described in great detail, including possible difficulties with application of numerical methods, and quantitative results can be easily reproduced. In most cases, numerical techniques are only necessary to evaluate integrals and minimize functions of one variable. As a rule, approximations of solutions by elementary functions are not given in the monograph. Although such approximations are widely used in the literature, it is believed that they are not efficient when there are several essential input parameters. For reasons of space, the main focus is on various highly undermatched tensile specimens, though undermatched and overmatched cases are briefly discussed as well.

Among the topics that are either new or presented in greater detail than would be found in similar texts are the following:

1. An approach to modifying upper bound solutions for a class of structures with no crack to account for the presence of a crack.
2. An approach to using singular velocity fields for constructing accurate upper bound solutions for highly undermatched joints.
3. The effect of the thickness of specimens on the limit load.
4. The effect of plastic anisotropy on the limit load.

5. A discussion of difficulties with application of numerical techniques in conjunction with simple kinematically admissible velocity fields.

**Chapter 1** concerns with the upper bound theorem for rigid perfectly plastic materials. A formal proof is not given because it can be found in any text on plasticity theory. Instead, original and efficient approaches to finding upper bound limit load solutions for welded joints with and with no cracks are introduced and explained. These approaches are used in subsequent chapters.

**Chapters 2–4** deal with highly undermatched specimens subject to tension. Firstly, in **Chap. 2**, two solutions for the center cracked specimen under plane strain conditions are presented. Each of these solutions illustrates one of the general approaches introduced in **Chap. 1**. The solutions found are generalized to scarf joint specimens in **Chap. 3**, also under plane strain conditions. Axisymmetric solutions are given in **Chap. 4**.

In **Chap. 5** two solutions for pure bending of highly undermatched panels under plane strain conditions are discussed. One of these solutions is based on an exact solution of plasticity theory. The other solution is obtained with the use of one of the universal methods proposed in **Chap. 1**. Comparison of the solutions determines the ranges in parametric space where each of them should be adopted.

**Chapter 6** includes a brief discussion of several topics. Firstly, the effect of the thickness of panels on the limit load is illustrated. To this end, the solution for the center cracked specimen presented in **Chap. 2** is compared to a new three-dimensional solution. The effect of the mis-match ratio is discussed next. Solutions for the undermatched and overmatched center cracked specimen are given and the definition for the highly undermatched case is clarified. Finally, it is shown that the effect of plastic anisotropy on the limit load is very significant and this material property should not be ignored in the development of flaw assessment procedures.

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# Symbols

The intention within the various theoretical developments given in this monograph has been to define each new symbol where it first appears in the text. A number of symbols are introduced in the abstract to individual chapters. These symbols re-appear consistently throughout the chapter. In this regards each chapter should be treated as self-contained in its symbol content. There are, however, certain symbols that re-appear consistently throughout the text. These symbols are given in the following list.

---

$F$	Tensile force
$F_u$	Upper bound on $F$
$f_u$	Dimensionless representation of $F_u$
$G$	Bending moment
$G_u$	Upper bound on $G$
$g_u$	Dimensionless representation of $G_u$
$u_x, u_y, u_z$	Components of kinematically admissible velocity field in Cartesian coordinates
$ \llbracket u_\tau \rrbracket $	Amount of velocity jump
$x, y, z$	Cartesian coordinates
$\zeta_{eq}$	Equivalent strain rate found using kinematically admissible velocity field
$\zeta_{xx}, \zeta_{yy}, \zeta_{zz},$ $\zeta_{xy}, \zeta_{xz}, \zeta_{yz}$	Components of kinematically admissible strain rate field in Cartesian coordinates
$\zeta_{eq}$	Equivalent strain rate
$\sigma_0$	Yield stress in tension
<b>n</b>	Unit normal vector
<b>U</b>	Velocity vector in rigid zone
<b>u</b>	Velocity vector in plastic zone

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