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Conformal Invariance: an Introduction to Loops, Interfaces and Stochastic Loewner Evolution

Springer

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Pour nos enfants:

Klara,
Léna, Ulysse, Ana-Fleur et
Alexandre Bonaventure

Preface

> "*Bien des choses ne sont impossibles que parce qu'on s'est accoutumé à les regarder comme telles.*"
>
> Charles Pinot Duclos (1751)

This volume was grown from lectures given at the atelier "*Modern applications of conformal invariance/Applications modernes de l'invariance conforme*" held in Nancy in march 2011. The enormous progress made possible by the systematic utilisation of methods of conformal field-theory for the precise understanding of critical phenomena, at least in two spatial dimensions, has been understood since almost three decades. Still, much of the impressive success of conformal invariance is related to *local* observables, their exponents and their correlators. Much recent work has been devoted to attempts to understand better the behaviour of *extended* objects, such as interfaces. This volume proposes an informal introduction to current results, methods and open questions, which we hope to be accessible to graduate students and researchers from other fields. Familiarity of the reader with the basic techniques in equilibrium statistical mechanics and critical phenomena is assumed.

In order to make this volume as self-contained as possible, we begin in Chap. 1 with a short and compact introduction to the main concepts and methods of $2D$ conformal invariance. We shall describe therein the main properties of *local* objects, such as primary scaling operators, the energy-momentum tensor, the Virasoro algebra, discuss unitary minimal models and how the partition function can be decomposed into minimal characters, both for bulk critical systems and for critical phenomena near a surface. Besides frequent references to spin systems known from statistical physics, such as the Ising and Potts models, we shall use the conformal field-theory of the free boson as the main paradigmatic example. Besides giving in this way a brief review of the textbook knowledge of those elements of conformal invariance which will be needed in the later chapters, we also introduce the notation to be used throughout this volume. Chapter 2, written by M. Bauer, gives an introduction to the physical and mathematical techniques required to describe an important class of growth models whose behaviour can be studied in great depth—conformally invariant interfaces governed by *Stochastic Loewner Evolution* (SLE). It turned out that the methods required for the study of SLE are quite distinct from those developed previously for the analysis of local observables within conformal field-theory, but they provide a very different and new point of view for the analysis of the behaviour of extended objects, and continue to fascinate physicists and mathematicians alike. Two of the most important properties of SLE, namely conformal invariance and the Domain Markov Property, are carefully explained and the chapter closes with a detailed discussion of the SLE-CFT correspondence. In carrying out this mathematically oriented analysis, a couple of technical assumptions had to be made. Although these may appear to be plausible, it is essential to verify whether these assumptions are actually realised in physically relevant systems. Chapter 3, written by C. Chatelain, presents a review of numerical tests of the basics of the SLE description of interfaces in critical systems. First discussing the most simple spin systems in the Ising and Potts universality classes, the second part of this review explores new ground in investigating to what extent SLE might become applicable in situations where quenched *disorder* becomes relevant. Finally Chap. 4, written by J.L. Jacobsen, addresses the study of two-dimensional loop models and their bulk and surface critical behaviour, analysed with the help of conformal invariance. After a detailed survey of the required graph-theoretical tools, it is shown how to relate the specific examples of the Potts- and $O(n)$-vector models in terms of clusters and oriented loops. In this way the defining model parameters, namely the number of states, can be analytically continued to arbitrary values. These analytic continuations are essential for application of the results to percolation and polymers. Since oriented loops act as level lines of height models, a treatment of these height models, via a geometric Coulomb gas construction, yields the bulk and surface critical exponents. From the underlying Temperley-Lieb algebra, the correct partition function in the continuum-limit can be found, which in turn is needed for the derivation for crossing formulæ in percolation. Novel extensions of the algebraic machinery of the Temperley-Lieb algebra appropriate for boundary conformal field-theory are explained.

Nous remercions chaleureusement les auteurs pour leur grand effort et leur temps devoué à l'écriture de ce volume. MH thanks the organisers of the Programme 'Advanced Conformal Field Theory and Applications' at the Institut Henri Poincaré in Paris for warm hospitality, where the editing was finished. It is a pleasure to thank Springer Verlag and especially C. Caron for having made the writing/editing process as *agréable* as possible.

La science vivante étant dans un processus de renouvellement permanent et fructueux, nous espérons évidemment que les générations futures exploreront les cieux qui pour nous seront restés des *mundi incogniti*. Nous dédions ce volume à nos enfants: à Klara, et à Léna, Ulysse, Ana-Fleur et Alexandre Bonaventure.

Nancy, France

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Abbreviations

Acronyms

BA	Bethe ansatz
BCFT	boundary conformal field-theory
CFT	conformal field-theory
CG	Coulomb gas
DLA	diffusion-limited aggregation
FK	Fortuin-Kasteleyn
IRF	interaction-around-a face
ISG	Ising spin glass
LERW	loop-erased random walk
OPA	operator-product algebra
OPE	operator-product expansion
RCFT	rational conformal field-theory
RFIM	random-field Ising model
RG	renormalisation group
SAW	self-avoiding walk
SLE	stochastic Loewner evolution
SOS	solid-on-solid
TL	Temperley-Lieb
$2D$	two-dimensional
$1D$	one-dimensional

Notations[1]

c	Virasoro central charge
κ	SLE parameter or 'diffusion constant'
B_t	standard Brownian motion
g_t	Loewner map
$L_n, \bar{L}_n$	generators of Virasoro algebra
$\Delta, \overline{\Delta}$	conformal weights
$\Delta_{r,s}$	elements of Kac table, conformal weights of minimal models
$x = \Delta + \overline{\Delta}$	scaling dimension
$s = \Delta - \overline{\Delta}$	spin
d_f	fractal dimension
$\mathbf{r}$	spatial coordinates (vector) in $2D$ with components $\mathbf{r} = r_1 + \mathrm{i}r_2 = x + \mathrm{i}y$
$z, \bar{z}$	complex coordinates
$T_{\mu\nu}$	components of energy-momentum tensor
$T(z), \overline{T}(\bar{z})$	complex energy-momentum tensor
$J(z), \overline{J}(\bar{z})$	conserved chiral current
ϕ, φ	generic symbol for conformal scaling operators
$\chi_{r,s}$	conformal character, of the primary operator $\phi_{r,s}$
V_α	$U(1)$ vertex operator, of charge α
τ	modular parameter of the torus
$P(q)$	generating function for the partitions of the integers related to Dedekind function $\eta(\tau) = q^{1/24} P(q)^{-1}$, with $q = e^{2\pi \mathrm{i}\tau}$
Z	partition function, functional integral
$\mathcal{L}$	Lagrangian density of classical field-theory
$\mathcal{H}$	classical hamiltonian (energy) of spin systems
H	quantum hamiltonian, logarithm of transfer matrix
$\langle . \rangle$	thermodynamic average
$\mathbb{Z}, \mathbb{R}, \mathbb{C}; \mathbb{H}$	integer, real & complex numbers; upper complex half-plane
$\mathcal{D}$	complex domain $\subset \mathbb{C}$
$\mathcal{T}$	tiling
Λ	lattice $\subset \mathbb{Z}^d$, with $\mathcal{N} = \lvert\Lambda\rvert$ sites
L	linear size of finite-size domains
$G, \mathfrak{g}$	Lie group and its associated Lie algebra, $\mathfrak{g} = \mathrm{Lie}(G)$

[1] Some notations commonly used in this book are listed in this section.