Part IV
Advanced topics

Part IV of this monograph continues the hierarchical projector based approach to DAEs, which is discussed in Part I, in view of three different aspects. We consider quasi-regular DAEs, nonregular DAEs, and ADAEs (abstract DAEs) in Hilbert spaces. An additional chapter conveys results obtained by the projector based analysis concerning minimization problems with DAE constraints.

The chapter on minimization starts with a discussion of adjoint and self-adjoint DAEs. It contains necessary and sufficient extremal conditions in terms of the original data. Special attention is directed to properties of the optimality DAE as the basis for indirect optimization methods. Further, an appropriate generalization of the Riccati feedback for LQPs is given.

For quasi-regular DAEs, we relax the constant-rank condition supporting the admissible matrix functions and the regularity. If, due to rank changes in a matrix function, a continuous nullspace no longer exists, we use instead a continuous subnullspace. In this way we figure out quasi-regularity. Linear DAEs that are transformable into standard canonical form are quasi-regular. However, the characteristic values characterizing regularity in Part I now lose their meaning and quasi-regularity appears to be somewhat diffuse—similarly to the differentiation index approach.

Nonregular DAEs may comprise a different number of equations and components of the unknown function. Discussing mainly linear DAEs, we emphasize the scope of possible different interpretations. We generalize the tractability index as well as the decouplings to apply also to those equations.

The concept of regular DAEs so far applied to DAEs in finite-dimensional spaces is then, in the chapter on ADAEs, generalized for equations

$$A(t)\frac{\mathrm{d}}{\mathrm{d}t}d(x(t),t)+b(x(t),t)=0,$$

with operators acting in Hilbert spaces. After having briefly discussed various special cases we turn to a class of linear ADAEs which covers parabolic PDEs and index-1 DAEs as well as couplings thereof. We treat this class in detail by means of Galerkin methods yielding solvability of the ADAE.