Part II
Index-1 DAEs: Analysis and numerical treatment

Part II constitutes a self-contained script on regular index-1 DAEs. It constitutes in essence an up-to-date improved and completed version of the early book [96]. While the latter is devoted to standard form DAEs, we now address DAEs of the form

$$f((D(t)x(t))', x(t), t) = 0,$$

with properly involved derivative.

This part starts with a chapter on the structural analysis of index-1 DAEs. It is shown that each solution of a regular index-1 DAE is actually a somewhat wrapped solution of an inherent explicit ODE. A certain decoupling function ω, resembling that in [96], plays its role. This inherent ODE is only implicitly given, but it is uniquely determined by the problem data. With this background, local solvability and perturbation results are proved.

In the chapter on numerical integration, backward differentiation formulas and certain classes of Runge–Kutta methods and general linear methods that are suitable for DAEs are discussed. Then we concentrate on the question of whether a given integration method passes the wrapping unchanged and is handed over to the inherent explicit ODE. The answer appears not to be a feature of the method, but a property of the DAE formulation. If the subspace $\operatorname{im} D(t)$ is actually time-invariant, then the integration method reaches the inherent explicit ODE unchanged. This makes the integration smooth to the extent to which it may be smooth for explicit ODEs. Otherwise one has to expect additional serious stepsize restrictions.

The third chapter addresses stability topics. Contractivity and dissipativity of DAEs are introduced, and it is discussed how integration methods reflect the respective flow properties. Again, one can benefit from a time-invariant subspace $\operatorname{im} D(t)$. Finally, stability in the sense of Lyapunov is addressed and the related solvability assertions on infinite intervals are allocated.